Book of Abstracts

Third International Workshop on Lot-Sizing

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Contents

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Programme

Time	Sunday 26 August 2012
19:00-21:30	**Welcome Dinner**

	Monday 27 August
8:30-9:00	Welcome and Registration
9:00-9:30	Opening Session
9:30-10:50	Incorporating Consumer Purchasing Behaviour on the Production Planning of Food Goods **Pedro Amorim**, Gonçalo Figueira, Alysson Costa and Bernardo Almada-Lobo
	A new heuristic for the non-stationary (s, S) policy **Onur A. Kılıç** and S. Armağan Tarım
10:50-11:20	Coffee Break
11:20-12:40	The Horizon Decomposition Approach for Capacity Constrained Lot Size Problems with Setup Times **Ioannis Fragkos**, Zeger Degraeve and Bert De Reyck
	Extending a Math-heuristic for Capacitated Lot Sizing towards a Hierarchical Solution Approach Marco Caserta and **Stefan Voß**
12:40-13:40	Lunch
13:40-15:40	An iterative two-phase heuristic approach for the Production Routing Problem **Nabil Absi**, Claudia Archetti, Stéphane Dauzère-Pérès and Dominique Feillet
	A Study of Integrated Lot Sizing and Cutting Stock Models for a Furniture Plant **Silvio A. de Araujo**, Socorro Rangel and Matheus Vanzela
	Integrated non-increasing capacitated lot sizing problem and preventive maintenance **Fahimeh Shamsaei** and Mathieu Van Vyve
15:40-16:10	Coffee Break
16:10-17:30	Capacitated lot sizing problem with inventory bounds **Ayse Akbalik**, Bernard Penz and Christophe Rapine
	The Capacitated Lot-Sizing Problem with Piecewise Concave Production Costs **Esra Koca**, Hande Yaman and M. Selim Akturk
17:30-18:30	Drinks

Time	Tuesday 28 August
9:00-10:20	Two-Period Convex Hull Closures for Big Bucket Lotsizing Problems **Kerem Akartunalı** and Andrew J. Miller
	Relaxations for two-level multi-item lot-sizing problem **Mathieu Van Vyve**, Laurence A. Wolsey and Hande Yaman
10:20-10:50	Coffee Break
10:50-12:50	Semidefinite relaxation of the DLSP with sequence-dependent changeover costs **Céline Gicquel**, Abdel Lisser and Michel Minoux
	An Integrated Approach for Solving Multi-Level Lot-Sizing and Scheduling Problems with Detailed Capacity Constraints Edwin David Gómez Urrutia, Riad Aggoune and **Stéphane Dauzère-Pérès**
	Modeling approaches to sequence dependent setups in lotsizing and scheduling **Luis Guimarães**, Diego Klabjan and Bernardo Almada-Lobo
12:50-13:50	Lunch
13:50-15:10	Complexity results for the single item uncapacitated lot-sizing problem with time-dependent batch sizes Ayse Akbalik and **Christophe Rapine**
	Lot-sizing with minimum batch sizes **Mathijn J. Retel Helmrich**, Wilco van den Heuvel, Raf Jans and Albert P.M. Wagelmans
15:30-16:30	Social event
17:00-18:00	Drinks
18:30-21:30	Conference dinner

Time	Wednesday 29 August
9:00-9:30	Discussion: Next meeting
9:30-10:50	Dynamic capacitated lot-sizing with parallel common setup operators **Karina Copil** and Horst Tempelmeier
	First Results on Multi-Level Capacitated Lot-Sizing in Closed-Loop Supply Chains **Florian Sahling** and Kristina Burmeister
10:50-11:20	Coffee Break
11:20-12:40	Linear programming models for the stochastic dynamic capacitated lot sizing problem **Timo Hilger** and Horst Tempelmeier
	Static-Dynamic Uncertainty Strategy for a Single-Item Stochastic Inventory Control Problem **Ulaş Özen**, Mustafa K. Doğrun and S. Armagan Tarim
12:40-13:40	Lunch
13:40-15:00	Effective Network Models for the Two-Level Serial Lot Sizing Problems Meltem Denizel, **Oğuz Solyalı** and Haldun Süral
	A Single Phase Dynamic Program with Independent Production Decision for Production-Capacitated Two- and Multi-Stage Lot-Sizing Problems **Hark-Chin Hwang**, Hyun-soo Ahn and Philip Kaminsky
15:00-15:30	Coffee Break
15:30-16:50	The Single-Item Green Lot-Sizing Problem with Fixed Carbon Emissions **Nabil Absi**, Stéphane Dauzère-Pérès, Safia Kedad-Sidhoum and Bernard Penz
	Bi-objective Economic Lot-Sizing models Edwin Romeijn, **Dolores Romero Morales** and Wilco van den Heuvel
16:50-17:50	Drinks

Foreword

At Erasmus University Rotterdam, we are very pleased to welcome you to the 3rd International Workshop on Lot-Sizing. The workshop has been organised after the excellent examples set by the Ecole Nationale Supérieure des Mines de Saint-Etienne in Gardanne, France (2010), and Özyeğin University in Istanbul, Turkey (2011).

The goal of the workshop is to cover recent advances in lot-sizing in order to facilitate the exchange of research ideas, promote collaboration among researchers from all over the world, and contribute to the further development of the field. With more than 20 presentations, the workshop programme shows that lot-sizing is indeed an active and diverse research area. With more than 40 participants, these presentations will unquestionably generate interesting discussions and stimulate new research ideas.

I would like to thank Wilco van den Heuvel who, with the assistance of Mathijn Retel Helmrich and Ursula David, did most of the organisational work for this workshop. We would also like to acknowledge the financial support by ERIM, the Tinbergen Institute and the Econometric Institute.

I wish you all a fruitful workshop and an enjoyable stay in Rotterdam.

Albert Wagelmans
Chair, Organizing Committee

Extended Abstracts

Incorporating Consumer Purchasing Behaviour on the Production Planning of Food Goods

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Abstract
In this research, we assess the impact of consumer purchasing behaviour on the production planning of perishable food products for companies operating in the fast moving consumer goods using direct store delivery. We build on previous marketing studies related to the effects of expiry dates, in order to derive mathematical formulae, which express the age dependent demand for different categories of perishable products. These demand expressions take into account both customer willingness to pay and product quality risk. A deterministic and a stochastic production planning model, which incorporate the customer's eagerness to pick up the fresher products available, are presented. Results point out that not considering the decreasing customer willingness to pay has an important impact both on the profit losses and on the amount of spoiled products. On the other hand, it was concluded that neglecting the fact that customers pick up the fresher products and the assumption that all products have the same production quality risk have a reduced impact on profit losses.

1 Introduction

The focus of this research is on the fast moving food consumer goods that are subject to physical spoilage. Examples of these products are found in the catering, dairy and processed food industries. The production systems related to these industries involve complex setup sequences that are often decided by specialized planners according to natural constraints. These characteristics together with high inventory rotation levels force the collapse of traditional tactical and operational planning levels [2]. Within this scope we consider that the producer (that ultimately sells perishable food products to final customers) has no control over the pricing of the products, which is assumed to be fixed within the considered planning horizon. In this setting, we propose mathematical models that are able to differentiate between different functions of the age dependent demand and/or between products with or without a stamped *best-before-date*. Our aim is to bridge the gap between consumer purchasing behaviour and production planning of perishable products by addressing the producers' problem arising from an increasing control over the downstream supply chain. This is indeed the case of many food industries that use direct store delivery and of companies that produce and sell in the same establishment (such as bakeries, for example). Hence, we extend the production planning formulations dealing with perishable products by incorporating the consumer purchasing behaviour. This is done by adjusting products' demand and inventory depletion to reflect the consumer's attitude towards perishability. We consider that demand is influenced by two distinct factors: the decreasing consumer willingness to pay for products with an increasing age and the different demand shapes that are related to the product quality risk. Moreover, we acknowledge that customers, having the opportunity to choose between equivalent products with different ages, will pick the fresher ones. In Figure 1 an example of a demand curve for lettuce throughout its shelf-life is presented.

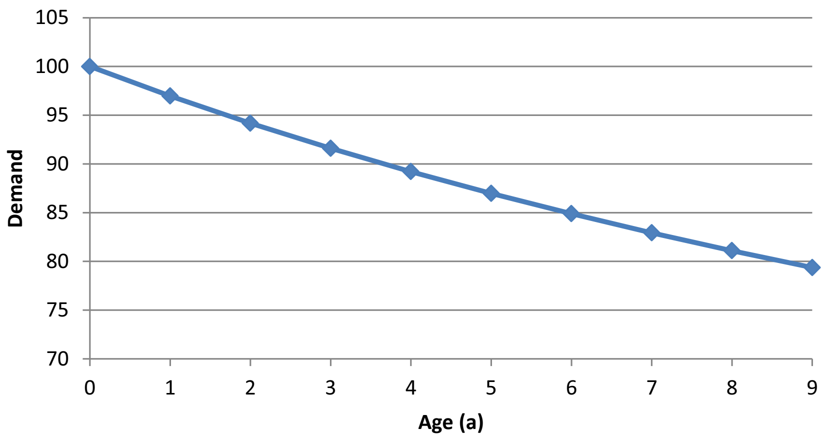

Figure 1: Example of the demand for lettuce over its shelf-life of 10 days, starting at 100 units.

2 Computational Study and Results

The computational study aims at understanding the impact of the consumer purchasing behaviour on the production planning of perishable food goods through several perspectives. Hence, we focus on:

1. the importance of considering age dependent demand as theorized in the consumer purchasing behaviour literature;

2. the importance of considering customer's eagerness to pick up the fresher products available;

3. the impact of neglecting different product quality risks, assuming a medium product quality risk for every product;

4. the amount of spoiled products due to not considering age dependent demand.

Overall the results point out that neglecting the age dependent demand resulted in a significant profit loss. As it is expected, more sensible customers yield higher losses and products with lower perceived risk, such as vegetables have a lower impact on the profit. Thus, if producers are able to reduce the perceived quality risk, this may have an important impact on the potential revenue. Most of these gains are achieved through an augmentation of the total demand throughout the product's shelf-life. It is also important to note that there is a considerable interaction between product quality risk and customer exigency. Hence, as customers get more sensible to perishability, the impact of product perceived risk tends to augment. Therefore, for example, if the producer is serving a retailer with very demanding customers, it is very important to decrease as much as possible the product quality risk (besides delivering very fresh products). In case of package food goods, such as yoghurt, this can be achieved by filling the product in glass containers instead of plastic ones [3]

Moreover, results point out that the profit loss of considering the fact that customer pick up fresher products is less significant than the loss coming from not considering the age dependent demand. In fact, while acknowledging the decreasing willingness to pay we are implicitly assuming consumers preference for fresher products. This will drive production plans towards a leaner strategy and, therefore, implicitly incorporate this characteristic of consumers. Hence, these results point out that despite the fact these constraints have a small impact on the profit loss, the solution structure may differ considerably as only the fresher products may be used to satisfy demand. Moreover, these constraints yield a greater reduction of the solution space for instances having products with longer shelf-lives. This is reflected in the higher profit losses.

Regarding the possibility of assuming a linear willingness to pay for all products. in all instance, a profit loss inferior to 1.3% was obtained. This indicates that differentiating between different production quality risk is not as important as differentiating between the remaining inputs for the age dependent demand. Our conclusion is that, in practical applications, planners of food products should focus on understanding both the initial willingness and the sensibility that customers have towards a decreasing shelf-life.

Finally, the results indicate that the amount of spoilage is severely impacted by the acknowledgement of a decreasing demand throughout the age of the product. The potential savings in product spoilage ascends to a complete reduction in the spoiled inventory. In an era of strong environmental awareness both in the civil society and in companies this is a crucial indicator to be taken into consideration.

3 Conclusions

In this study we first develop a set of age dependent demand functions for products with different product quality risks based on researches analysing consumer purchasing behaviour for perishable food products. We propose a deterministic model for the production planning of perishable goods that accounts both for decreasing willingness to pay and customers' eagerness to choose the product in a fresher state. This deterministic model is extended to a stochastic one dealing with demand uncertainty, which is a common characteristic of the fast moving food consumer goods markets. The computational study focuses on a sensitivity analysis where the main parameters related to the novelties introduced are varied. Results pointed out that extending food production planning models to deal with an age dependent demand is of great importance both in terms of profit and product spoilage.

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A new heuristic for the non-stationary (s, S) policy

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In recent years, more industries have been experiencing non-stationary (time-varying) stochastic demand with higher frequency. This can mainly be attributed to the fact that product life cycles are getting increasingly shorter in response to fast technological progress and rapid changes in consumer preferences. The life cycle of a product typically involves a series of phases, and demand rate changes radically when product moves from one phase to another. This immediately suggests that demand is never stationary. However, when product life cycle spans a relatively short period of time, the extent of non-stationarity becomes drastic since demand rate changes very rapidly within time. Furthermore, in most environments demand is often heavily seasonal and has a significant trend. Therefore, demand rate also changes within the phases of product life cycle. In this context, firms must employ inventory control policies which can effectively handle non-stationary demand to match their supply to demand [3].

Managing inventories is more demanding when demand is non-stationary. The difficulty mainly springs from the fact that fluctuations in demand must be reflected in replenishments. In other words, non-stationary demand necessitates non-stationary inventory control. Notwithstanding, probably the most challenging non-stationary inventory control problems arise when replenishments require fixed freight fees. This is often the case for companies using distant offshore suppliers. Here, the non-stationarity of demand affects not only the size but also the timing of replenishments. This leads to a very sophisticated inventory problem where replenishment decisions must be dynamically determined considering possible future demands as well as impending inventory needs.

This paper addresses the inventory control problem in a finite-horizon periodic-review system with non-stationary stochastic demands, fixed replenishment costs, and linear holding and backorder costs. The relevance of the problem is evident, since it appears in many retail, wholesale, and industrial environments. The problem is well addressed in the literature. It is known that the non-stationary (s, S) policy is optimal to control inventories [5]. The (s, S) policy is a min-max policy characterized by a re-order level – s_n and an order-up-to level – S_n for each review period n throughout the planning horizon. At the beginning of period n, if the inventory position is below s_n, then a replenishment order is placed for a quantity that will bring the inventory position up to S_n.

Despite the structure of the optimal policy has already been established, finding the optimal parameters of the policy still remains a challenging problem [4]. This paper seeks to address this issue. We propose a new heuristic to compute near-optimal parameters of the non-stationary (s, S) policy. The heuristic relies on the idea of establishing policy parameters by making use of individual cost functions associated with prospective replenishment cycles. This allows us to take advantage of convexity, and compute near-optimal policy parameters by using a dynamic programming algorithm. The proposed algorithm is very efficient since the state space of the dynamic program is independent from possible demand realizations and inventory positions.

We conduct an extensive computational study involving a variety of cost parameters and demand characteristics. We mainly follow the numerical setting in [2], and in addition we make use of the empirical demand patterns reported in [3]. We design three sets of experiments which are respectively based on seasonal, trend and empirical demands patterns. We compare the new heuristic with the ones developed in [1] and [2], as well as the optimal dynamic program. The results show that the new heuristic significantly outperform the earlier heuristics proposed in [1] and [2]. It yields an average optimality gap of 0.1% for seasonal and trend patterns, and 0.5% for the empirical pattern.

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The Horizon Decomposition Approach for Capacity Constrained Lot Size Problems with Setup Times

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Abstract
We introduce the horizon decomposition, a novel application of Dantzig-Wolfe decomposition in the context of the capacity constrained lot size problem with setup times. The problem horizon is partitioned in contiguous overlapping time intervals that create subproblems identical to the original problem, but of smaller horizon length. The master problem includes linking constraints that determine the amount of shared information across the subproblems. By introducing a family of reformulations of the original problem, the user has the flexibility to determine the subproblem size and the master size almost independently. An empirical investigation of different combinations of master and subproblem sizes gives insights on how to select an efficient horizon partition for various problem classes. Computational experience with a branch-and-price algorithm shows that the horizon decomposition approach is competitive or outperforms state of the art branch-and-cut solvers, depending on the problem structure. Finally, we show how this methodology can be extended to problems with generic constraint structure.

1 Introduction

The capacity constrained lot size problem with setup times (CLST) is a natural extension of the single-item uncapacitated lot size problem studied by Wagner and Whitin in their seminal paper [1]. The extension involves multiple items which share common time capacity in each period. Each item consumes time capacity for setup and for production. Given a fixed time horizon, a feasible production plan satisfies (i) the per item demand requirements of each period (ii) the setup restrictions and (iii) the time capacity restrictions. The goal is to find the production plan that minimizes the inventory holding cost and setup cost of all items in the given horizon.

This paper introduces a novel way of applying Dantzing-Wolfe decomposition in lot sizing problems and demonstrates its application to the CLST. By introducing linking constraints for a set of variables, the problem is decomposed in several subproblems that may share common periods and are short horizon CLSTs. The computational benefit of this approach does not stem from the specialized structure of the subproblem but from the reduction of its size. Since the subproblems do not have the integrality property, the resulting lower bound is better than that of the LP relaxation [2]. Two important characteristics of the horizon decomposition approach is that (i) it can be applied to more general classes of lot sizing problems and (ii) the user can control the size of the master problem and of the subproblem with two parameters which are almost independent.

2 Computational Experiments

A computational study is utilized in order to investigate three main research questions. Initially, the focus is on determining the best performing combination of subproblem size and master problem size, in terms of CPU time and lower bound quality. The main finding is that a small number of overlapping periods is often sufficient to drastically improve the lower bound quality. Large overlaps result in degenerate master programs which induce poor convergence of column generation and deliver a small bound improvement. Moreover, the number of items and the horizon length of each instance can be used to determine a more informed horizon partition. Second, we focus on how useful this idea is in the development of heuristic approaches. Specifically, we utilize a column generation algorithm that applies the relaxation induced neighborhood search heuristic at termination [3]. This approach outperforms the best heuristic found in terms of average gap quality, while it is competitive in finding heuristic solutions. Finally, we develop a branch-and-price algorithm that we test in a new challenging set of instances that we generated. For problems with few items, the horizon decomposition approach greatly outperforms branch-and-cut. As the number of items increases it remains competitive and it consistently delivers better lower bounds.

3 Generalizations

The principle of horizon decomposition can be viewed as a particular application of Lagrange decomposition [4]. However, it is different from lagrange decomposition in two ways. First, it does not consider existing sets of constraints whose combinatorial nature is "easy" but it rather creates new linking constraints and the resulting subproblems are of the same structure. Second, the user has the flexibility to select among a family of different reformulations. We demonstrate how this idea can be generalized to mixed integer linear programs with generic structure. Two extensions are presented. First, we consider non-overlapping sets of constraints and partition the original problems by creating copies of certain variables. In this formulation,the subproblems are defined over mutually exclusive sets of constraints. Second, an extension is presented that introduces new variables and partitions existing constraints in such a way that the subproblems have the same set of constraints, but different variables. We discuss potential applications of each formulation.

4 Conclusions

This work introduces the principle of horizon decomposition in the context of lot sizing problems. A implementation on the CLST and a computational study show the potential benefits of this approach. The computational study gives empirical evidence on which horizon partitions are efficient for certain problem classes and investigates the trade-off between lower bound quality and CPU time. A heuristic approach based on horizon decomposition outperforms the best approach found in the literature in terms of integrality gap. Moreover, a branch-and-price algorithm is competitive or outperforms state of the art branch-and-cut software in terms of integrality gaps, while it almost always delivers a better lower bound. Finally, two potential extensions of this principle on generic MIPs are presented.

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Extending a Math-heuristic for Capacitated Lot Sizing towards a Hierarchical Solution Approach

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Abstract
The multi-item multi-period capacitated lot sizing problem with setups (CLST) is a well known optimization problem with wide applicability in real-world production planning problems. Based on a recently proposed Dantzig-Wolfe approach we have presented a novel math-heuristic algorithm for the CLST. In this paper, we provide reasoning that the approach may be beneficial when additional constraints like perishability constraints are added. From an algorithmic standpoint this also constitutes an important extension when looking at it from the point of view of solution methods. Adding a hierarchy of related constraints may be seen as defining specialized problems to be solved. In doing so we may even benefit when going back to solving the original problem.

1 Introduction

In recent years, a lot of attention has been devoted to the integration, or hybridization, of metaheuristics with exact methods. This exposition also relates to the term math-heuristics [4] describing works which are, e.g., exploiting mathematical programming techniques in (meta)heuristic frameworks or on granting to mathematical programming approaches the cross-problem robustness and constrained-CPU-time effectiveness which characterize metaheuristics.

Based on a recently proposed Dantzig-Wolfe approach of [2], in [1] we presented a novel math-heuristic algorithm for the CLST. The major contribution of this paper lies in the presentation of an algorithm exploiting exact techniques (Dantzig-Wolfe) in a meta-heuristic fashion. To measure the effectiveness of the proposed approach, we have tested the algorithm to solve standard benchmark instances from [6]. Here we extend in two directions. We add considerations with respect to perishability or hop constraints. And we add very interesting observations on hierarchical relaxations within our approach exemplified by means of those constraints.

2 The CLST

The Multi-Item Multi-Period Capacitated Lot Sizing Problem with Setups (CLST) is a well known problem finding a wide variety of real-world applications. The CLST is $\mathcal{NP}$-

hard [6]. A mixed-integer formulation of the CLST is:

$$
\begin{aligned}
\text{(CLST): min} \quad z = \quad & \sum_{j=1}^{n}\sum_{t=1}^{T}\left(f_{jt}y_{jt} + c_{jt}x_{jt} + h_{jt}s_{jt}\right) + \sum_{j=1}^{n} h_{j0}s_{j0} \\
\text{s.t.} \quad & \sum_{j=1}^{n}\left(a_{jt}x_{jt} + m_{jt}y_{jt}\right) \leq b_t \quad \forall t \\
& s_{jt-1} + x_{jt} = d_{jt} + s_{jt} \quad \forall j,t \\
& x_{jt} \leq M y_{jt} \quad \forall j,t \\
& y_{jt} \in \{0,1\} \quad \forall j,t \\
& x_{jt}, s_{jt} \geq 0 \quad \forall j,t
\end{aligned}
$$

where items $j = 1, \ldots, n$ should be produced over time periods $t = 1, \ldots, T$. In the CLST formulation, f_{jt}, c_{jt}, and h_{jt} indicate the fixed cost, the unitary production cost and the unitary inventory holding cost for item j in period t, respectively. Parameters m_{jt} and a_{jt} indicate the setup time and the unitary production time, respectively, while b_t stands for the production capacity in period t. Parameter d_{jt} indicates the demand of item j in period t. Finally, in the model, three sets of decision variables are employed, *i.e.*, $y_{jt} \in \{0,1\}$, which takes value 1 if there is a setup for item j in period t; as well as $x_{jt} \geq 0$ and $s_{jt} \geq 0$, indicating the production volume and the inventory level for item j in period t, respectively. Note that s_{j0} is given as data indicating initial inventory.

Due to its vast industrial applicability, researchers have devoted special attention to the CLST. Since the CLST is still difficult to solve to optimality, many researchers have tried to tackle the problem by working on relaxations of the same. A good description of some well studied relaxations of the CLST is provided by [5]. A recent discussion of solution approaches for the CLST can be found in [3].

3 Solution Approaches

Starting from an observation of [2], in [1] we presented a Dantzig-Wolfe reformulation in which the setup variables and the production variables are dealt with separately. More specifically, for any given setup plan, we generated columns in which the production plan needed not be a dominant plan, *i.e.*, we introduced into the master problem columns corresponding to solutions in which, for certain periods, there might be a setup without having a production. Thus, a single setup plan induces a number of non-dominant production plans. However, due to the large size of the set of non-dominant plans induced by each setup plan, we designed a mechanism inspired by the corridor method to bound the search of non-dominant production plans in the neighborhood of the Wagner-Whitin dominant plan associated to the current setup plan. By adding an exogenous constraint to the pricing problem, we collected a set of new columns and, subsequently, we priced into the master problem those columns with negative reduced costs. Finally, the column generation approach was repeated until no new columns with negative reduced costs were found.

In this paper, the controlled addition of hop constraints is explored to become a hierarchical relaxation scheme. Table 1 presents some results, in terms of solution quality and running time, of the use of CPLEX 12.1 over the problem obtained after the inclusion of hop constraints into the standard CLST model (i.e., the Dantzig-Wolfe algorithm has

Instance		Hop constraints value T'					Cum Time	CPLEX
		1	2	3	4	5		
Tr6-15	z^*	40155	38599	37767	37721	-	-	37721
	T	0.49	3.84	0.59	1.28	-	6.21	1.24
Tr6-30	z^*	62979	61784	61746	-	-	-	61746
	T	0.06	29.1	62.9	-	-	92.1	126.2
Tr12-15	z^*	80904	76052	74752	74702	74634	-	74634
	T	0.54	5.1	3.35	5.54	3.05	17.7	2.67
Tr12-30	z^*	142097	131673	130600	130596	-	-	130596
	T	0.44	49.71	45.16	136.8	-	261.92	154.41
Tr24-15	z^*	142847	137506	136678	136509	-	-	136509
	T	1.04	4.97	18.9	5.4	-	30.31	23.70
Tr24-30	z^*	302472	289099	287929	-	-	-	287929
	T	0.49	23.18	76.67	-	-	100.34	110.99

Table 1: Results on six instances from [6] with incremental use of hop constraints.

even not been used in this phase of the computations; for results with the Dantzig-Wolfe algorithm see [1]). In the table, the first column presents the instance name. For each instance, we record the optimal value obtained by the MIP solver along with the running time (see column CPLEX; all times on a dual core pentium 1.8GHz Linux PC). The header of each column indicates the value of T', *i.e.*, the maximum number of time periods an item is allowed to remain in inventory. For example, considering instance `Tr6-15` within the column with a hop constraint value of 1, we observe that the optimal solution when the hop constraints are defined for $T' = 1$ has a value of 40155, and the running time of CPLEX 12.1 is of 0.49 seconds. Columns regarding a hop constraint value of 2 to 5 provide similar information for increasing values of T'. Thus, we iteratively solved the problem each time enhancing the hop constraint, *i.e.*, increasing the value of T', until the known optimal value of the original CLST is obtained. In the case of instance `Tr6-15`, the known optimal value of 37721 was obtained when $T' = 4$. Thus, the iterative process stops at $T' = 4$, since larger values of T' would lead to the same optimal solution. Note that in the current settings we know the optimal values and our primary interest is in investigating the behaviour of hierarchically changing the hop constraint. This changes once we investigate on how to avoid solving each model with a slightly modified hop constraint from scratch and also incorporate a stopping criterion regarding extending the hierarchy.

Column `Cum Time` of Table 1 presents the cumulative running time of the iterative process. That is, this column shows the total running time for all values of T' until the known optimal solution is reached for the first time. Again, with respect to instance `Tr6-15`, the cumulative running time of the iterative approach is 6.21 seconds, while CPLEX reaches the optimal solution for the original CLST in 1.24 seconds. Interestingly enough the cumulative running times may even be smaller than those of CPLEX when no hop constraints are considered at all (this is the case for instances `Tr6-30` and `Tr24-30`).

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An iterative two-phase heuristic approach for the Production Routing Problem

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Abstract
In this paper we investigate the integrated optimization of production, distribution and inventory decisions in a simple supply system, where retailers have to be restocked from a central production facility. We propose an iterative two-phase heuristic approach to solve our problem. The main advantages of our approach are its simplicity and efficiency. Some experimental results are reported to show the effectiveness of our method and its limits.

1 Introduction

We investigate the integrated optimization of production, distribution and inventory decisions in a simple supply system, where retailers have to be restocked from a central production facility. A single-item uncapacitated lot-sizing problem is defined for representing production in the facility and inventory management. Daily distribution is modeled using a vehicle routing problem framework. Such a problem is challenging since it integrates two decisions, namely: lot-sizing and vehicle routing.

The problem of integrating production and routing decisions was introduced by [4]. Most of authors used heuristic methods to solve the problem of integrating production planning and vehicle routing ([3], [1]), while few authors used exact methods ([2]). In this paper we propose an iterative two-phase heuristic method that iteratively focuses on the lot-sizing or distribution parts of the system.

We consider a set $M = \{1, \ldots, |M|\}$ of retailers and a single product sold by these retailers along a discrete time horizon $T = \{1, \ldots, H\}$. Consumption rate for retailer $i \in M$ during time period $t \in T$ is denoted r_{it}. Retailers can be restocked from a common production facility. Products are then kept in retail stores, with an inventory limit U_i and at a unitary cost h_{it}, dependent of the retailer and of the time period. The central production facility is identified with index 0. No limit is assumed on production. A maximal inventory level U_0 of finished products is however defined. Unitary inventory costs in the facility are assumed to be constant over time and are denoted h_0. A fixed production cost K and a variable cost p proportional to the number of items produced, are also considered at each producing period for the facility. A set of vehicles $V = \{1, \dots, |V|\}$ of capacity C is considered for distribution. Travel costs c_{ij} are defined between every pair (i, j) of locations, including the production facility and the retailers. We note $A = M \cup \{0\} \times M \cup \{0\}$ the set of these pairs. No limit is imposed on vehicle tour duration.

The aim of the problem is to simultaneously optimize production, inventory and routing so that final demands of customers and inventory limits in production facility and retailers are satisfied, while minimizing all types of costs.

2 An iterative two-phase method

We propose to solve our problem using an iterative two-phase method. The general scheme of the method is presented in Algorithm 2 where *sol* stores the best solution found so far.

Algorithm 1 General scheme of the two-phase method

```
sol ← ∅
Initialize SC*_vit for each i ∈ M, v ∈ V and t ∈ T
repeat
   repeat
      // Intensification phase
      Solve the Lot-sizing Problem (SC*_vit) and get γ*_vit for each i ∈ M, v ∈ V and t ∈ T
      Solve the Routing Problem (γ*_vit)
      Update sol if necessary
      Update SC*_vit
   until a stopping criterion is met
   Update SC*_vit // Diversification phase
until a stopping criterion is met
```

In the first phase of the method, routing costs incurred when visiting a retailer i at a given period t using a vehicle v are approximated and denoted SC_{vit}. When the SC_{vit} are fixed, the initial problem can be transformed into a Lot-sizing Problem that aims at optimizing production and inventory levels. Distribution costs only interfere with this model in the form of setup costs SC_{vit}, taken into account in the value of the solution when retailer i is replenished at period t using vehicle v. From now on the first phase will be called *Lot-sizing Problem* (SC_{vit}).

On the basis of the solution obtained in the first phase, routing decisions are taken in a second phase. The second phase will be called *Routing Problem* (γ_{vit}) and corresponds to the solution of the routing problem once the decision variables (γ_{vit}) to visit retailers are fixed, thus once we know the set of retailers to be visited at each period.

Fixed costs SC_{vit} play a central role in this approach as they create a connection between the first and the second phase. The value of SC_{vit} is initially set to $c_{0i} + c_{i0}$. This will force the solution of the *Lot-sizing Problem* (SC_{vit}) to serve less frequently those retailers which are far away from the supplier and, thus, for which the corresponding transportation cost is high. However, the initial value of SC_{vit} does not take into account the clustering of retailers: There is no measure of the proximity among retailers visited in a certain day, so retailers that are very far away from each other may be clustered together and served in the same day. This of course has a very bad impact on the transportation cost. Thus, in subsequent iterations, the values of SC_{vit} are updated using the information provided by the solution of the *Routing Problem* (γ_{vit}) so that solutions of the *Lot-sizing Problem* (SC_{vit}) are driven to better solutions in terms of retailer clustering.

Once this procedure converges, and instead of using a multi-start procedure, we use the SC_{vit} multipliers to diversify local optimum solutions. The SC_{vit} multipliers are updated according to the best known solution. The goal is to help the procedure exploring neighbors that are not explored during the intensification phase.

3 Experimental results

In order to evaluate the efficiency and robustness of our algorithm, we perform experiments using 480 instances from Archetti et al. [2]. These instances are classified in 3 sets (A1, A2 and A3) and are characterized by 6 time periods and respectively 14, 50 and 100 customers with constant demand, no production capacity and no plant inventory capacity, but with initial inventory at the plant and customers. The instance set A1 has a single capacitated vehicle and sets A2 and A3 have unlimited number of capacitated vehicles. We compare our approach (denoted IM) to the heuristic (denoted H) proposed by Archetti et al. [2] and the one proposed by Adulyasak et al. [1] (denoted ALNS1000). In what follows we summarize our experimental results by presenting the average gap to the best known solution for each method (Table 1). Table 2 reports the number of time our method outperforms H and ALNS1000 (denoted respectively #BetterH and #BetterALNS1000).

Set of instances	H	ALNS1000	IM
A1	2,99%	1,79%	0,29%
A2	1,104%	0,56%	0,206%
A3	0,85%	0,30%	0,35%

Table 1: Average gaps from best known solutions

Set of instances	#BetterALNS1000	#BetterH	Total number of instances
A1	96	96	96
A2	57	88	96
A3	44	70	96

Table 2: Number of best solutions

Tables 1 and 2 show that our method give very good results comparing to H and ALNS1000. The computing time of our method is relatively high since we use a standard solver to solve the lot-sizing part (about 100 seconds for instances with 14 customers, about 300 seconds for instances with 50 customers and about 1000 seconds for instances with 100 customers).

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A Study of Integrated Lot Sizing and Cutting Stock Models for a Furniture Plant

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Abstract
The integrated lot sizing and cutting stock problem is an optimization problem that considers simultaneously decisions related to the lot sizing problem and to the cutting stock problem. The goal is to capture the interdependence between these decisions in order to enable economy of raw materials and reduction of production costs and inventory. In this work we intend to give an overview of models that consider the integrated cutting-stock and lot-sizing problem on a furniture factory. The objective is to identify their abilities and limitations in describing relevant aspects of this production environment. A new model is proposed and results of a preliminary computational study are presented and discussed.

1 Introduction

The Furniture industry in Brazil, although spread around the country, is concentrated in regional centers, mostly situated in the south and southeast regions. Each regional center includes companies of different sizes and specialties. This study is motivated by a small scale furniture factory (thereafter called Plant V) located in the northwest region of the Sao Paulo state and included in the regional center of Votuporanga.

The Plant V is specialized in the production of bedroom furniture which is manufactured using wooden plates of different sizes and types. The production system at Plant V is very similar to that of other plants in the region. At the beginning of the week the production manager decides which types of furniture and how many will be produced during that week. The production line is then fully dedicated to these products. At first (cutting stage) the rectangular wooden plates in stock (plates) are divided into rectangular smaller pieces (pieces) that will compose a given type of furniture. The pieces are then manually processed according to the product design and pass through several other stages (e.g. gluing, drill, painting) before they are grouped to compose a final product, packed (mounted or not) and stored. There is not much space for storage of the final product, nor to store the pieces that will not be processed during the working day.

The process of cutting the plates may involve loss of material, that is, pieces that are cut and are not part of the demand. The factory is interested in reducing these losses given that they have a strong impact in the costs of the final product. One way of reducing these losses is increasing the types of demanded pieces and their demand. More pieces types may allow a better arrangement of the pieces in the plate (cutting pattern). Moreover, increasing the pieces demand might help to reduce the number of setups due to the fact that more plates may be cut simultaneously with the same cutting pattern. All this can be achieved if the industry anticipates the production of some final products. However, the anticipation of production may incur in additional inventory costs. To capture all these elements in the decision process, cutting stock and lot sizing, a combined decision should be taken.

2 Contribution

Few papers have considered the integrated cutting-stock and lot-sizing problem in several industrial sectors (*e.g.* [2], [8], [9], [10], [11]). In the furniture industry we highlight the papers from [7], [6], [5], [1], [12], [4], [3] and [13]. The models presented in [1], [12], [3] and [13] were developed considering the production process of Plant V. In this work the mathematical models for integrating both decisions in the context of Plant V are presented and their differences and similarities highlighted. Moreover some new contributions will be presented including a mathematical model for which the column generation technique is used to solve the associated linear relaxation. The results of a preliminary computational study, conducted using data collected at the plant, indicate that it is possible to reduce the total cost of inventory and raw materials when the planning is done in an integrated manner.

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Integrated non-increasing capacitated lot sizing problem and preventive maintenance

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1 Extended Abstract

In the current competitive world, technological advancement does not guarantee that the machineries do not break down during their life. A machine's condition can turn into an out-of-control condition gradually or at once. This implies that the products would not have their potential perfect quality. Besides, such situation questions the validity of the utilized lot sizing model, in that the portion of defective products in each lot increases. The probability of shifting to an out-of-control condition will be diminished by scheduling periodic inspection and performing cyclic/non-cyclic maintenance.
The objective of maintenance scheduling is to achieve the best time to perform the maintenance actions in order to minimize the total maintenance cost. On the other hand, the objective of production planning is to minimize the total production and inventory costs while satisfying the demand for all products. Since production planning and maintenance scheduling are interdependent they cannot be planned sequentially, though many companies do it. Albeit the resultant more complicated model, the synchronization between production planning and maintenance scheduling induces significant reduction in the total cost incurred by production interruptions, delays, and re-planning. During the last few decades, exhaustive research has been carried out in the realm of lot sizing problem and EMQ (Economic Manufacturing Quantity) as a classic model has been exploited to develop more complicated realistic models by relaxing its underlying assumptions.
Many of the researchers in the literature review have considered cyclic (or periodic) maintenance in their preventive maintenance models [Aghezzaf et al. (2007)]. In periodic PM we deal with optimizing how often each task in a set of predefined tasks should be executed. There is a relatively limited literature on models presenting a general (not necessarily periodic) preventive maintenance policy [Fituhi and Nourelfath (2012), Aghezzaf and Najid (2008)]. The objective of these models is to determine either the best time for doing preventive replacements by new items, i.e., perfect PM [Yao et al. (2004)],or the optimal sequence for imperfect maintenance actions [Levitin and Lisnianski (2000)]. The authors' survey of the literature did not discover a study that deals, in particular, with an efficient formulation for noncyclic PM and the effect of production interruptions as a result of PM on the lot sizing problem. In this paper we are going to fill this research gap by developing some efficient models. The existing works in the literature investigate that, albeit consuming a fraction of the nominal capacity, how PM can be exploited to

decrease the number of defective products. In spite of the similarity of our work to to the existing works, in terms of consuming a fraction of the nominal capacity by PM, we assume that all products always have the same quality regardless of when PM is carried out. This is a realistic assumption in, for example, lime industry whereby our work is inspired.
In this paper we consider the integrated problem of cyclic (and noncyclic) preventive maintenance and production planning for a single machine. We are given a set of products that must be produced in lots on a capacitated production system during a specified finite planning horizon. The capacity is decreasing over time, unless a preventive maintenance is peformed. The amount of decrease might depend on the production level or the number of set-ups peformed. The proposed model determines simultaneously the optimal production plan and the timing of preventive maintenance actions. The integrated problem is used to compare the cyclic and non-cyclic maintenance policies and the value of using non-cyclic preventive maintenace is illustrated through a numerical example.

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Capacitated lot sizing problem with inventory bounds

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Abstract
We consider the capacitated lot sizing problem (CLSP) with bounded inventory. We show that the single item CLSP with inventory bounds is polynomial for a stationary production capacity and under the Wagner-Whitin cost structure. The multi-item CLSP with a dedicated capacity for each item type and a common capacitated storage shared between the different items is proven to be strongly NP-hard.

1 Introduction

The CLSP consists in planning production and storage activities with a minimum cost, in order to satisfy the deterministic demand over a finite horizon under a limited production capacity. Different extensions are largely studied in the literature (see Brahimi *et al.* [3]). In the classical CLSP, the storage capacity is assumed to be illimited, which is clearly not realistic in some cases. See for instance Atamtürk and Küçükyavuz [2] or Akbalik *et al.* [1] for some emerging industrial applications with limitated storage. In the literature, the single item uncapacitated lot sizing problem (ULSP) with inventory bounds is largely studied (see Table 1), but to the best of our knowledge, the problem with a capacitated production is not yet explored.

Table 1: The ULSP with time-dependent inventory bounds

Articles	Cost structure	Backlogging	Complexity
Love [7]	piecewise concave	yes	$\mathcal{O}(T^3)$
Gutiérrez *et al.* [4]	concave	yes	$\mathcal{O}(T^3)$
Toczylowski [10]	linear	no	$\mathcal{O}(T^2)$
Atamtürk and Küçükyavuz [2]	linear (+fixed holding cost)	no	$\mathcal{O}(T^2)$
Sedeno-Noda *et al.* [9]	linear (no setup cost)	no	$\mathcal{O}(Tlog(T))$
Hwang and van den Heuvel [5]	concave	yes	$\mathcal{O}(T^2)$
Hwang and van den Heuvel [5]	linear and WW	yes	$\mathcal{O}(T)$

Multi-item and multi-echelon extensions of the bounded inventory problem are also studied by different authors. In Jaruphongsa *et al.* [6], the authors consider a two-echelon ULSP with delivery time windows and warehouse capacity constraint. Minner [8] considers a problem of replenishment of multi-item in a warehouse having a limited capacity and compares the performance of three heuristics. There exists also some polyhedral studies on the extensions of the bounded storage problem that we do not cite here.

In this study, we consider both capacitated production and capacitated storage activities. We present an $\mathcal{O}(T^3)$ time algorithm for the single item CLSP with a stationary production capacity, time-dependent storage capacities and under Wagner-Whitin cost structure. The second extension studied is multi-item CLSP with a stationary production capacity for each item type and a stationary storage capacity shared by the different items. The multi-item problem we study differs thus from the classical multi-item lot sizing problem, where the production capacity is shared by different items, which we do not consider.

2 Single Item Capacitated Lot Sizing

We consider the single item capacitated lot sizing problem with inventory bounds (CLSP-IB). Demands are known over a time horizon of T periods and are to be satisfied on time (backlogging is not allowed), either from the stock on hand or from the production at the current period. A setup cost f_t is paid at period t if a production occurs, in addition to a unit production cost p_t per unit produced. A unit holding cost h_t is also charged to carry a unit in stock from period t to $t+1$. We restrict to non-speculative motives, also called Wagner-Whitin (WW) cost structure, which imposes that $p_t + h_t \geq p_{t+1}$ for any period $t < T$.

In this article, we consider a capacity both on the production and the storage. More precisely, let x_t be the quantity produced at period t and s_t the stock level at the end of period t. We require that for each period t, x_t does not exceed a given value P_t, and that s_t does not exceed a given value H_t. Recall that the lot sizing problem with time-varying capacity is known to be $\mathcal{NP}$-hard, even with no inventory bounds ($H_t = +\infty$). On the contrary, we establish that CLSP-IB remains polynomially solvable for a stationary production capacity P and time-varying inventory bounds H_t. For this, following Hwang and van den Heuvel [5], we generalize the classical notion of regeneration point to the periods where one of the stock constraints $0 \leq s_u$ or $s_u \leq H_{u-1}$ is saturated :

Definition 1 *A period u is said to be an inventory period if its entering stock level is either* 0 *or* H_{u-1}.

Given a planning, a subplan (u, v) corresponds to the sequence $(u+1, \ldots, v)$ of periods such that u and v are inventory periods. We also call a period t *fractional* if its production is neither 0 nor at full capacity (that is $0 < x_t < P$). Assuming a WW-cost structure, it is easy to prove the following classical dominance :

Property 1 *There exists an optimal planning such that each subplan contains at most one fractional period. If the fractional period exists, it is necessary the first period of the subplan.*

The first part of the dominance still holds without non-speculative assumptions, but the fractional period may eventually appear inside the subplan. Using a dynamic programming approach, we derive the following result :

Theorem 1 *The single item CLSP-IB with a stationary production capacity can be solved in time $\mathcal{O}(T^3)$ under the WW cost structure.*

3 Multi-item Capacitated Lot-Sizing

For the multi-item case, we consider a particular problem where the inventory space is the only shared resource. More specifically, we consider that we have m different types of item, with specific demands to satisfy for each item over the time horizon. Each item i is produced on a dedicated machine, with a stationary capacity. To this point we have m independent single item lot-sizing problems to deal with, and thus the problem could be solved in polynomial time. However all the items produced share the same storage facility, which, due to physical limitations, can not hold more than H units at any point in time. We have the following result :

Theorem 2 *The multi-item CLSP-IB with item dedicated machine is $\mathcal{NP}$-hard in the strong sense.*

The polynomial reduction is performed from the EXACT COVER BY 3 SETS problem (X3C). The problem remains $\mathcal{NP}$-hard even with no storage cost, no unitary production cost and a stationary setup cost.

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The Capacitated Lot-Sizing Problem with Piecewise Concave Production Costs

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Abstract
In this study, we consider the single item capacitated lot sizing problem (CLSP) with piecewise concave costs for which the computational complexity of the problem is an open question. We present a polynomial time algorithm for the CLSP with piecewise concave production costs when the number of breakpoints of the cost function is fixed and these breakpoints are the same for all periods. A piecewise concave function can be used to represent discounts, subcontracting, capacity acquisition and overloading decisions. Therefore, the problem we will study and the algorithm we will present can be applicable to many production systems.

1 Introduction

Although the CLSP is studied since 1960s, most of the studies consider the problem with concave production costs and there are only few studies on the CLSP with piecewise concave production costs in the literature. Among these studies, Lippman [1] assumes that the length of the interval between any two consecutive breakpoints of the production cost function is the same for all periods and he proposes a polynomial time algorithm. Swoveland [2] presents an important characteristic of an optimal solution to the problem which we will make use of in our solution approach. For his solution method, he assumes that the breakpoints of the production cost function may vary between periods but all the breakpoints have a common divisor and his algorithm is pseudo-polynomial in this common divisor, so in the breakpoint levels. Chen et al. [4] and Shaw and Wagelmans [3] consider the CLSP with piecewise linear production costs, and in both of these studies the algorithms given are not polynomial. The algorithms proposed in any of these studies cannot be used to solve the problem that we will consider in polynomial time. Therefore, computational complexity of the problem we will study is an open question and we will show that it is polynomially solvable.

2 The Dynamic Programming Algorithm

The single item CLSP with piecewise concave costs can be defined as follows. There is a planning horizon of n periods for a single item. The demand for the item in period t is $d_t \geq 0$ for $t = 1, \ldots, n$. For each period t, the cost of producing x_t units is equal to $p_t(x_t)$, and holding s_t units during period t costs $h_t(s_t)$. Production capacity of period t is C_t units.

We assume that the piecewise concave function $p_t(.)$ has m_t breakpoints, $b_t^1, \ldots, b_t^{m_t}$ with $b_t^0 = 0$ and $b_t^{m_t} = C_t$, $b_t^{i-1} < b_t^i$, and $h_t(.)$ is a concave function on $[0, \infty)$. We obtain the following property for a regeneration interval.

Property 1 *There exists an optimal solution to the problem CLSP such that in each regeneration interval $[j, l]$, there exists at most one period period i with $j \leq i \leq l$ such that $x_t = b_t^z$ for some $z \in \{0, \ldots, m_t\}$ for all $t \in \{j, \ldots, l\} \setminus \{i\}$, and $x_i \in [b_i^0, b_i^{m_i}] \setminus \{b_i^0, \ldots, b_i^{m_i}\}$. We will call such a period as a fractional period.*

This property is similar to the one in Swoveland [2]. By using Property 1, we can find a minimum cost solution for the regeneration interval $[j, l]$. We will develop a dynamic programming (DP) algorithm for solving the problem.

In order to develop the DP, we assume that the breakpoint levels of the production cost function are the same for all periods, i.e, $b_t^i = b_i$ for all $t = 1, \ldots, n$ and $i = 0, \ldots, m$ where $m_t = m$ for all $t = 1, \ldots, n$. We assume that the functions p and h can be evaluated in constant time.

Let $\tau \in Z_+^m$ and $t \in \{j, \ldots, l\}$. We define $F_{jl}(t, \tau)$ to be the minimum cost for periods j up to t during which τ_i times b_i, for $i = 1, \ldots, m$, units are produced, no fractional production is done, $s_{j-1} = s_l = 0$ and $s_u > 0$ for $u \in \{j, \ldots, \min\{t, l-1\}\}$.

For $i = 0, \ldots, m$, we evaluate $F_{jl}(j, e_i)$ and let $F_{jl}(j, \tau) = \infty$ if $\sum_{i=1}^m \tau_i \geq 2$. For $t \in \{j+1, \ldots, l\}$, and $\tau \in Z_+^m$, we compute $F_{jl}(t, \tau)$ using the following recursive formula.

$$F_{jl}(t,\tau) = \begin{cases} \infty & \text{if } \sum_{i=1}^m \tau_i > t-j+1 \text{ or} \\ & (\sum_{i=1}^m \tau_i b_i \leq d_{jt} \text{ and } t < l) \text{ or } (\sum_{i=1}^m \tau_i b_i \neq d_{jl} \text{ and } t = l); \\ \min_{i=0,\ldots,m:\tau \geq e_i} \left\{ F_{jl}(t-1, \tau - e_i) + p_t(b_i) + h_t\left(\tau^T b - d_{jt}\right) \right\} & \\ & \text{otherwise.} \end{cases}$$

For given t and τ, $F_{jl}(t, \tau)$ can be computed in $O(m)$ time. As $\tau_i \leq n$ for $i = 1, \ldots, m$, we have $O(n^m)$ possible τ vectors. For a given interval $[j, l]$, the function F_{jl} can be evaluated in $O(mn^{m+1})$ time.

Let $\tau \in Z_+^m$ and $\pi \in Z_+^{m-1}$. If τ_i times b_i, for $i = 1, \ldots, m$, units are produced in periods j up to $t-1$ and π_i times b_i, for $i = 1, \ldots, m-1$, $\left\lfloor \frac{d_{jl} - \sum_{i=1}^m \tau_i b_i - \sum_{i=1}^{m-1} \pi_i b_i}{b_m} \right\rfloor$ times b_m units are produced in periods $t+1$ to l, then the fractional production amount $\rho_{jl}(\tau, \pi)$ that is produced in period t is equal to

$$\rho_{jl}(\tau, \pi) = d_{jl} - \sum_{i=1}^m \tau_i b_i - \sum_{i=1}^{m-1} \pi_i b_i - \left\lfloor \frac{d_{jl} - \sum_{i=1}^m \tau_i b_i - \sum_{i=1}^{m-1} \pi_i b_i}{b_m} \right\rfloor b_m.$$

Finally, we define $G_{jl}(t, \tau, \pi)$ to be the minimum cost for periods j up to t during which τ_i times b_i units, for $i = 1, \ldots, m$, and one time a fractional production is done (in any period) given that π_i times b_i, for $i = 1, \ldots, m-1$, and $\left\lfloor \frac{d_{jl} - \sum_{i=1}^m \tau_i b_i - \sum_{i=1}^{m-1} \pi_i b_i}{b_m} \right\rfloor$ times b_m units are produced after period t. Because of the page limit, we do not give the necessary formulas for calculating this function.

For given t, τ and π, $G_{jl}(t, \tau, \pi)$ can be computed in $O(m)$ time. Therefore, G_{jl} can be evaluated in $O(mn^{2m})$ time.

We can find the minimum cost for the interval $[j, l]$ by

$$\mu_{jl} = \min_{\tau \in \{0,\dots,n\}^m} \left\{ F_{jl}\left(l, \tau\right), G_{jl}\left(l, \tau, e_0\right) \right\}.$$

Once μ_{jl}, for $1 \leq j \leq l \leq n$, are computed we can solve the problem by solving a shortest path problem in a graph we will construct. As μ_{jl} can be computed in $O(mn^{2m})$ time and there are $O(n^2)$ intervals, we require $O(mn^{2m+2})$ time to construct the graph. This dominates the time to compute a shortest path. Therefore, the overall complexity is $O(mn^{2m+2})$.

We develop this algorithm for the CLSP with piecewise concave costs and constant capacities. The dynamic programming algorithm has an important implication. By this algorithm, we show that the CLSP with piecewise concave costs can be solved in polynomial time when the number of breakpoints of the cost function (m) is fixed. Moreover, with small modifications we can use the algorithm for solving different cases of the problem. For example, we can solve the problem when backordering is allowed without changing the computational complexity. Moreover, if there is no production capacity for any period, i.e., $b_m = C_t = \infty$, for all t, we adapt the DP so shat the overall complexity is $O((m-1)n^{2m+1})$. Atamturk and Hochbaum [5] consider a special case of this problem in which they assume $m = 2$. They solve the problem in $O(n^5)$ time, For $m = 2$, our algorithm also solves the problem in $O(n^5)$ time. Lastly, if we consider the problem studied by Hellion et al. [6], our algorithm also solves the problem in the same time order with their algorithm.

We implement the proposed DP algortihm in Java. In order to compare the algorithm's performance we first assume that the production cost function is piecewise linear and we implement the incremental formulation of CLSP with piecewise linear cost functions in XPressMP. In Table 1, a sample of our computational study can be seen. We are currently investigating how different MIP formulations, such as incremental, multiple choice or convex combination models, might affect the computation times.

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n	m	DP	MIP
10	2	0.062	0.078
20	2	2.215	3.729
20	2	2.153	4.04
30	2	29.25	217.194
30	2	28.626	197.103
48	2	680.848	999.811*
48	2	674.114	999.858*

* Terminated due to 1000 sec time limit

Table 1: Comparison of the DP with an MIP formulation

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Two-Period Convex Hull Closures for Big Bucket Lotsizing Problems

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Abstract
Despite the significant attention they have drawn, big bucket lotsizing problems remain notoriously difficult to solve. We discuss results indicating that what make these problems difficult are the embedded single-machine, single-level, multi-period submodels. We therefore consider the simplest such submodel, a multi-item, two-period capacitated model. We propose a methodology that would approximate the closure of this submodel by generating violated inequalities using a distance function. We present computational results that indicate how useful knowing the structure of these two-period models could be in solving more complicated problems.

1 Introduction

In spite of extensive research, focus of the mathematical programming community has been mainly on single-item problems, with some of the single-item problem results extended to multi-item problems. There are some recent studies providing insight into some versions of capacitated multi-item problems, and [5, 6] are rare studies investigating the big bucket problems. Previous computational results in the literature have indicated high duality gaps for big bucket lotsizing problems, even though some strategies can be partially efficient for generating lower bounds and feasible solutions. The study accomplished in [1] has provided us important feedback on why big bucket lotsizing problems are still very hard to solve. More specifically, better approximations for the convex hull of the single-machine, single-level, multi-period capacitated problems are necessary to accomplish better results on general lotsizing problems. In this paper, we investigate the potentials of the simplest such model, a two-period model. In order to accomplish this, we propose a methodology that does not require reformulating the problems a priori or predefine valid inequalities. It is important to remark that we are not aware of strong inequalities or reformulations for these subproblems. Therefore, we also have the motivation to use the proposed framework to achieve these characteristics.

In a recent study, [3] approaches the single-item capacitated lot-sizing problem by formulating it as a bottleneck flow network problem, allowing the authors to define facet-defining inequalities. The extension of this specific formulation to multi-level structure is a motivation for the simple formulation we use in our framework. Another motivation for the proposed framework is that two-period problems are computationally very easy to solve, as experienced in previous relax-and-fix based heuristics.

In the last decade, there have been a big body of research and promising results on the "closures" of general cutting planes and some particular polyhedrons, with even partially achieving elementary closures helping to close duality gaps efficiently. We can define "closure" as the smallest possible polyhedron that includes all the valid inequalities of a type. In our context, we generate valid inequalities for the convex hulls of two period subproblems and hence approximate their closures, and therefore, we use the terminology "Two-period Convex Hull Closure". More specifically, our approach cuts off fractional points of a general lotsizing problem using the characteristics of the convex hull of a two-period submodel, where column generation is used to generate the extreme points of these convex hulls, and Farkas' Lemma provides the basis for validity of these cutting planes. To our knowledge, this is a new methodology in the lotsizing literature, although similar approaches can be found in the integer programming literature, e.g. the "local cuts" idea of [2] mapping a fractional solution of TSP into a lower dimension and searching a cut separating it.

2 The Basic Idea

We define the feasible region of the two-period relaxation, referred to as X^{2PL}:

$$x^i_{t'} \le M^i_{t'} y^i_{t'} \qquad i = [1, ..., NI], t' = 1, 2 \tag{1}$$

$$x^i_{t'} \le \widetilde{d}^i_{t'} y^i_{t'} + s^i \qquad i = [1, ..., NI], t' = 1, 2 \tag{2}$$

$$x^i_1 + x^i_2 \le \widetilde{d}^i_1 y^i_1 + \widetilde{d}^i_2 y^i_2 + s^i \qquad i = [1, ..., NI] \tag{3}$$

$$x^i_1 + x^i_2 \le \widetilde{d}^i_1 + s^i \qquad i = [1, ..., NI] \tag{4}$$

$$\sum_{i=1}^{NI} (x^i_{t'} + ST^i y^i_{t'}) \le \widetilde{C}_{t'} \qquad t' = 1, 2 \tag{5}$$

$$x, s \ge 0, y \in \{0, 1\}^{2xNI} \tag{6}$$

The variables x^i_t and y^i_t represent the production and the binary setup decision of item i in period t, and s^i represents the inventory at the end of two periods. The vector M consists of sufficiently big numbers, and C and ST indicate capacity and setup times for various items and time periods. On the other hand, the parameter $\widetilde{d}$ represents the remaining cumulative demand, e.g., $\widetilde{d}^i_1$ is the demand for i from period 1 onwards.

This submodel is a multi-item extension of the bottleneck flow formulation of [3] (when $NT = 2$) and it also extends the single-period study of [5, 6]. Since we consider only one stock variable per item, a time subscript t is not necessary for these variables. Note that the constraints (2) and (3) are simply the (ℓ, S) inequalities of [4].

Given a fractional solution $(\overline{x}, \overline{y}, \overline{s})$, one can define the infinity-norm distance ($\mathcal{L}_\infty$) of this solution to $conv(X^{2PL})$ using the extreme points of X^{2PL}. As the number of extreme points can grow exponentially, column generation is used to generate the favorable extreme points of $conv(X^{2PL})$. Using these favorable extreme points, we check whether a given fractional solution can be written as a convex combination or not. If not, we can generate a valid inequality using Farkas' Lemma that cuts off the fractional point. Note that this local cut is in the convex hull closure of this two-period relaxation. Finally, note that alternative distance functions such as $\mathcal{L}_1$ or even $\mathcal{L}_2$ can be used in the framework, and this important aspect will be discussed in detail in the presentation, including the strength of the cuts generated.

One remark is that this framework is not based on predefining a family of valid inequalities, which is one of its advantages. An inequality will be generated in all cases when the fractional solution is not in the convex hull of a two-period problem. This is also the justification for our expectation that this framework will provide an adequate approximation of the bottleneck of the general lotsizing problems, as this is focused on the capacitated single-machine problems with an approach providing exact solutions for the subproblems. Computational results considering two-period as well as realistic-size problem instances will be discussed in detail in the presentation.

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Relaxations for two-level multi-item lot-sizing problem

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Abstract
We consider several variants of the two-level lot-sizing problem with one product/item at the upper level facing dependent demand, and multiple products/items/clients at the lower level, facing independent demand. We first show that under a natural cost assumption, it is sufficient to optimize over a stock-dominant relaxation. We further study the polyhedral structure of a strong relaxation of this problem involving only initial inventory variables. We consider several variants: uncapacitated at both levels with or without start-up costs, uncapacitated at the upper level and cosntant capacity at the lower level, constant capacity at both levels. We finally demonstrate how the strong formulations described improve our ability to solve instances with up to several dozens of periods and a few hundred products.

We study two-level multi-item multi-period planning problems on a finite horizon with time-dependent demand. In this context, multi-level means that there is dependent demand in the system: some goods are consumed by the production of others. We focus on problems with one product/item at the upper-level facing dependent demand, and multiple products/items/clients at the lower level, facing independent demand. The two levels can represent different stages of a production process executed at a single location (e.g making and packing, bulk and end products, component and assembly), but also represent production and transportation to client area(s), in which case it is known as the one warehouse, multiple retailer (OWMR) problem. One key aspect of the models that we consider is that holding inventory is possible at both levels. We study various polyhedra related to such problems. In particular, we consider the uncapacitated problem, the problem with start-up cost at both levels, and some capacited variants.

For single-item lot-sizing, many polyhedral results have been obtained, for the basic uncapacitated model [2, 1]and for extensions including backlogging, [4, 3], start-ups [8], constant capacity [5], increasing capacities [7], sales, or a combination of these [10]. These results can be classified into two categories: linear description of the convex hull of solutions in the original variable space, usually of exponential size and accompanied by an efficient separation algorithm on the one hand, and tight extended formulation involving additional variables, usually of polynomial size on the other hand. For the latter, Van Vyve and Wolsey [9] show how to create and manage a trade-off between strength and size of these extended formulations.

To the best of our knowledge, no polyhedral work has been done for multi-level lot-sizing models involving start-ups, capacities, or multiple items at the lower level (beyond single-item relaxations based on the echelon stock concept). The present work partially fills this gap. Following Pochet and Wolsey [6], we consider stock-dominant relaxations of these multi-level problems that we prove are sufficient to solve the problem under specific cost assumptions.

We first describe the capacitated two-level lot-sizing model 2LS, its stock-dominant relaxation 2WW and the closely related subproblem 2DLS whose polyhedral structure we study in order to obtain a good formulation for 2WW. We prove that solving 2WW solves 2LS under a natural cost assumption.

Next we consider several variants of 2DLS and describe tight formulations, both extended and in the original variable space. We first consider the basic uncapacitated 2DLS-(U,U) model and give a polynomial-size LP extended formulation, together with its projection onto the original variable space. We then, sometimes partially, extend these results in several directions. The main result is that the following formulation

$$s_0^0 = \sum_{i \in I} \phi^i, \tag{1}$$

$$\phi^i + s_0^i = \sum_{l=1}^{n} d_l^i \zeta_l^i \qquad i \in I, \tag{2}$$

$$s_0^i = \sum_{l=1}^{n} d_l^i \delta_l^i \qquad i \in I, \tag{3}$$

$$\zeta_l^i \geq \delta_l^i \qquad i \in I, l \in [1,n], \tag{4}$$

$$\zeta_l^i + y_{1t}^0 + y_{t+1,l}^i \geq 1 \qquad i \in I, l \in [1,n], t \in [0,l], \tag{5}$$

$$\delta_l^i + y_{1l}^i \geq 1 \qquad i \in I, l \in [1,n], \tag{6}$$

$$\zeta \in \mathbb{R}_+^{mn}, \delta \in \mathbb{R}_+^{mn}, y \in \mathbb{R}_+^{(m+1)n}, \tag{7}$$

$$\zeta \in \mathbb{Z}_+^{mn}, \delta \in \mathbb{Z}_+^{mn}, y \in \mathbb{Z}_+^{(m+1)n}. \tag{8}$$

is a tight extended formulation for for the two-level discrete lot-sizing problem 2DLS:

$$s_0^0 = \sum_{i \in I} \phi^i, \tag{9}$$

$$\phi^i + s_0^i + My_{1t}^0 + \sum_{j=t+1}^{l} Q_j^i y_j^i \geq d_{1l}^i \qquad i \in I, l \in [1,n], t \in [0,l], \tag{10}$$

$$s_0^i + \sum_{j=1}^{l} Q_j^i y_j^i \geq d_{1l}^i \qquad i \in I, l \in [1,n], \tag{11}$$

$$s_0 \in \mathbb{R}_+^{m+1}, y \in \{0,1\}^{(m+1)n}, \phi \in \mathbb{R}_+^m. \tag{12}$$

The proof is by showing integrality of the associated polyhedron.

We also consider the model 2DLS-(U,U)-SC that includes start-ups and extend the result obtained for 2DLS-(U,U). We also derive results for the case with constant capacity limits on production of items at the lower level, and at both levels respectively. Finally we demonstrate how these strong formulations improve our ability to solve several variants of two-level planning problems. We also indicate what are the best modelling optiions for instances of very large sizes. We conclude by discussing some open problems.

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Semidefinite relaxation of the DLSP with sequence-dependent changeover costs

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Abstract
We study the DLSP with sequence-dependent changeover costs. We formulate the problem as a quadratic binary program and propose to compute a lower bound of its optimal solution value by using a semidefinite relaxation of the problem. The results of our computational experiments show that the proposed approach provides lower bounds of significantly improved quality as compared with those provided by the best previously published linear relaxations.

1 Introduction

We study the multi-product single-resource discrete lot-sizing and scheduling problem with sequence-dependent changeover costs or DLSPSD. As defined in [4], several key assumptions are used in the DLSPSD to model the production planning problem:
- A set of products is to be produced on a single capacitated production resource.
- A finite time horizon subdivided into discrete periods is used to plan production.
- Demand for products is time-varying (i.e. dynamic) and deterministically known.
- At most one product can be produced per period and the facility processes either one product at full capacity or is completely idle ("discrete" production policy).
- Costs to be minimized are the inventory holding costs and the sequence-dependent changeover costs.

We first introduce a quadratic binary programming formulation for the DLSPSD.
Notation and decision variables
- $p = 1...P$: products (product $p = 0$ represents the idle state of the machine),
- $t = 1...T$: periods (period $t = 0$ represents the initial state of the system),
- D_{pt}: demand for p in t expressed as a multiple of the production capacity,
- h_p: inventory holding cost per unit per period for product p,
- S_{pq}: cost of a changeover from product p to product q.
- y_{pt}: decision variables, $y_{pt} = 1$ if product p is assigned to period t, 0 otherwise.

QBP formulation

$$Z_{DLSP} = \min \sum_{p=1}^{P} \sum_{t=1}^{T} h_p \sum_{\tau=1}^{t} (y_{p\tau} - D_{p\tau}) + \sum_{p,q=0}^{P} S_{p,q} \sum_{t=0}^{T-1} y_{pt} y_{qt+1} \quad (1)$$

$$\forall p, \forall t, \quad \sum_{\tau=1}^{t} y_{p\tau} \geq \sum_{\tau=1}^{t} D_{p\tau} \quad (2)$$

$$\forall t, \quad \sum_{p=0}^{P} y_{pt} = 1 \quad (3)$$

$$\forall p, \forall t \quad y_{pt} \in \{0, 1\} \quad (4)$$

The objective function (1) corresponds to the minimization of the inventory holding and changeover costs over the planning horizon. (2) are the demand satisfaction constraints. Constraints (3) ensure that a single product is assigned to each period of the planning horizon.

Previously published exact solution approaches to solve the DLSPSD are mostly based on the linearization of the quadratic terms of the objective function through the introduction of linearization variables (see e.g. [1]). The obtained MILP formulation can then be further strengthened e.g. thanks to the use of an extended reformulation (see [3]). However, even if substantial improvements of the lower bounds can be obtained by strengthening the MILP formulation, there are still cases where the linear relaxation of the DLSPSD is of rather weak quality. This is why we propose to compute lower bounds by using a semidefinite relaxation of the problem. To the best of our knowledge, this is the first attempt at explicitly using a QBP formulation to solve the DLSPSD and at using semidefinite programming to solve lot-sizing problems.

2 Semidefinite relaxation of the DLSPSD

We first carry out the reformulation of the DLSPSD (1)-(4) as a semidefinite program. This is done mainly by relying on techniques discussed in the semidefinite programming literature for general (0-1) quadratic binary programs (see e.g. [5]). A key ingredient in this step is the representation of the linear constraints of the original problem in the quadratic space. Namely, a significantly better semidefinite relaxation can be obtained by applying a pretreatment to the linear (equality or inequality) constraints before reformulating them in the SDP (see e.g. [5] and [6]).

The obtained semidefinite program cannot be solved as such due to the presence of some nonconvex constraints in its formulation. We thus have to carry out a relaxation of the problem: we enlarge the feasible set to make it convex by dropping the nonconvex constraints. This leads to a polynomially solvable convex optimization problem but as some constraints of the original problem have been removed, the corresponding optimal solution value will only provide a lower bound on the integer optimal solution value Z_{DLSP}.

This initial semidefinite relaxation of DLSPSD can finally be strengthened thanks to the use of several families of valid inequalities, either developped specifically for the problem under study (see [7]) or proposed to strengthen semidefinite relaxations of general quadratic binary programs (see e.g. [5]). The number of these valid inequalities grows very fast with the problem size so that it is not possible to include all of them directly in the semidefinite programming formulation. This is why we devised a standard cutting-plane generation algorithm to include them as needed in the formulation.

	Set	1	2	3	4	5	6	7	8	9	10
Instance size	P	4	6	4	6	4	4	6	4	6	4
	T	15	15	20	20	25	15	15	20	20	25
LP relaxation	Gap (%)	1.9	0.3	1.3	2.1	1.4	11.2	4.2	7.2	7.5	7.2
	Time (s)	0.1	0.1	0.2	0.2	0.2	0.1	0.1	0.1	0.2	0.2
SDP relaxation	Gap (%)	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.0	0.2
	Time (s)	42	86	151	644	713	95	145	388	852	1196

Table 1: Computational results

3 Computational results

We randomly generated 10 sets of 10 small instances involving 4 to 6 products, 15 to 25 periods, with a production capacity utilization of 95%. The instances mainly differ with respect to the structure of the changeover cost matrix S. Instances of sets 1-5 have a general cost structure whereas instances of sets 6-10 correspond to the frequently encountered case where products can be grouped into product families.

For each instance, we compute:
- the optimal integer solution value Z_{ip}.
- the lower bound Z_{ext} provided by the linear reformulation of (1)-(4) discussed in [1]. We then use the extended formulation of the single-product DLSP proposed in [3] to strengthen its continuous relaxation.
- the lower bound Z_{sdp1} provided by the strengthened SDP relaxation of the QBP formulation proposed in section 2.

Z_{ip} and Z_{ext} are computed using CPLEX 12.1 solver, Z_{sdp1} is computed thanks to the semidefinite programming solver DSDP (see [2]). All tests were run on an Intel Core i5 (2,7 GHz) with 4 Go of RAM, running under Windows 7.

Table 1 displays the computational results: we provide the average gap between the lower bounds (Z_{ext} or Z_{sdp1}) and Z_{ip} as well as the average computation time needed to compute these lower bounds.

Results from table 1 show that the lower bounds provided by the proposed semidefinite relaxation of the DLSPSD are of significantly improved quality as compared with the ones provided by the strongest linear relaxations known for the problem. Namely, for instances with a general changeover cost structure, the average gap over the 50 instances is decreased from 1.4% with the extended linear reformulation to 0.04% with the semidefinite relaxation. The improvement is more significant for instances with a product family cost structure as the average gap over the 50 corresponding instances is decreased from 7.5% to 0.04%. However, the computation time needed to obtain the semidefinite lower bounds remains high. This is mainly due to the fact that a series of semidefinite programs of increasing size has to be solved by the cutting-plane generation algorithm. This could be improved to some extent thanks to the use of a warm-start strategy within the cutting-plane generation. However, as explained in [5], this would be difficult to implement with interior-point algorithms such as the one embedded in solver DSDP. It might thus be worth investigating the use of other algorithms such as the spectral bundle method to solve the semidefinite programs involved in the proposed approach.

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An Integrated Approach for Solving Multi-Level Lot-Sizing and Scheduling Problems with Detailed Capacity Constraints

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1 Introduction

Lot-sizing and scheduling problems are usually studied separately, by viewing lot sizes as inputs/restrictions of the scheduling problem. There exist in the literature some problems which consider sequencing decisions in lot sizing models. These problems are the *Discrete Lot-sizing and Scheduling Problem* (DLSP), the *Continuous Setup Lot-sizing Problem* (CSLP), the *Proportional Lot-sizing and Scheduling Problem* (PLSP) and the *General Lot-sizing and Scheduling Problem* (GLSP) (see [4]). The first three problems consider small time buckets while the last problem considers big time buckets. The major drawback of these problems is that, even if solutions are feasible, the capacity is not efficiently used because of the limitations on the number of setups per period and the *all-or-nothing* assumption. In addition, these restrictions increase the number of discrete variables, thus making the problem very difficult to solve. This is why these problems usually cover simple manufacturing systems (with one or two parallel resources and few items). Another well-known problem dealing with capacity considerations is the *Capacitated Lot-Sizing Problem* (CLSP), where the planning horizon consists of big time buckets and there are not limitations on the number of setups per period or on how the the available capacity is used. Nevertheless, sequencing constraints are not taken into account, and thus the resulting production plans may be unfeasible at the scheduling level, when studying manufacturing systems with more than one resource. To avoid this unfeasibility problem and to efficiently use capacity, we propose an integrated approach for multi-level lot-sizing problems. Our mathematical model is a generalization of the *Multi-Level Capacitated Lot-Sizing Problem* (MLCLSP), using detailed capacity constraints and the echelon stock formulation described in [2]. This work is an extension of the integrated approach for single-level problems presented in [5], which is an improved version of the original approach proposed in [8] and [1]. A previous integrated approach was proposed in [3], where the lot sizing and scheduling problems are solved iteratively and separately, so that the output of one problem is the input for the other one.

2 Integrated approach

The proposed approach considers a multi-item lot-sizing mathematical model, as in the MLCLSP, but in which aggregate capacity constraints are replaced by detailed capacity constraints, allowing multi-resource problems to be solved by considering typical scheduling restrictions, like precedence between operations of the same job and precedence between operations sharing the same resource. Multi-level considerations are modeled by using flow balance and Bill-Of-Material (BOM) constraints in an echelon stock formulation. The objective is to minimize the sum of manufacturing, setup and echelon stock costs. The model allows determining an optimal production plan for a fixed sequence of operations. Then, a sequence improvement method is applied and a new sequence is fixed and used to determine a new production plan.

The scheduling problem is modeled with a disjunctive graph, where nodes correspond to operations and arcs represent precedence constraints between operations. Arcs linking operations of a same job always have the same orientation (respecting the routing of items), but the orientation of arcs linking operations on the routing of resources has to be decided. Each path in the resulting conjunctive graph corresponds to a capacity constraint. The last operation of each path must be finished before its due date in order to meet the corresponding deadline, i.e. the end of the associated period in the planning horizon.

2.1 Multi-level lot-sizing problem with a fixed sequence of operations

Even for a fixed sequence of operations, the number of paths in the conjunctive graph is huge. Therefore, in order to make the problem solvable and to use the Wagner-Within property, Lagrangian relaxation using subgradient multipliers is applied, relaxing the capacity and BOM constraints. Then, solving the relaxed problem corresponds to solving a set of uncapacitated single-item lot-sizing problems (USILSP), using the algorithm proposed in [7]. As there are too many capacity constraints, explicitly managing all of them is impossible. Thus, only the most violated capacity constraint is relaxed at each iteration of the Lagrangian relaxation algorithm, by only updating the Lagrangian multiplier associated to the most critical path. This method provides at each iteration a lower bound to the original multi-level multi-item lot-sizing problem with detailed capacity constraints and a fixed sequence, which corresponds to the total cost of a production plan which is in most cases unfeasible. This is why a smoothing procedure is implemented, that moves quantities of some items from some periods to others, in order to satisfy the relaxed constraints while trying to avoid increasing too much the overall production plan cost. If a feasible plan is obtained, its cost is an upper bound of the integrated problem. As moving quantities to satisfy capacity constraints may increase the violation of BOM constraints and vice versa, one of the challenges of the smoothing procedure is to balance between decisions that improve the satisfaction of the two types of constraints. An extended description of the solution approach for a fixed sequence can be found in [6].

2.2 Sequence improvement method

Solving the lot-sizing problem for a fixed sequence allows generating a possibly optimal feasible plan for that sequence. Nevertheless, this plan could be improved by modifying the sequence. To search for an optimal integrated plan, i.e. with minimum total cost and also the best possible use of the capacity, our approach integrates a sequence improvement method using a Tabu search heuristic. The idea is to search for a new conjunctive graph, according to the information of the Lagrangian relaxation on the most violated paths. Moves are performed by swapping critical arcs in the original conjunctive graph, and evaluated by applying the Lagrangian heuristic for the resulting conjunctive graphs. The conjunctive graph, i.e. the sequence, providing the best feasible plan is selected, and the swapped arc is added in the Tabu list.

3 Numerical results

Currently, numerical experiments have only been performed for the case with a fixed sequence. Instances for a job-shop manufacturing system with 6 (resp. 10) jobs, 6 (resp. 10) operations per job and 6 (resp. 10) resources have been solved, varying the number of periods (5, 10 and 20) of the planning horizon and the capacity per period. Results were compared with optimal solutions provided by the standard solver IBM ILOG CPLEX (version 12.3). The quality of the solutions of our approach is good (with a maximum gap of 1.95% over 13 instances) and the computational times are shorter than those of CPLEX. Numerical results for the integrated multi-level lot-sizing and scheduling problem will be presented in the workshop.

4 Conclusions

An integrated approach for solving multi-level lot-sizing problems with detailed scheduling constraints was introduced. First numerical results allow validating the method for the case with a fixed sequence, and additional tests are being conducted to show the effectiveness of the proposed integrated approach (with sequence improvements). More complex instances will also be considered. Our future research aims at including in the model practical constraints encountered in a supply chain context.

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Modeling approaches to sequence dependent setups in lotsizing and scheduling

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Abstract
Several production environments require simultaneous planing of sizing and scheduling of production lots. Integration of these problems has received an increased attention from the research community due to its inherit applicability to real world problems. A two-dimensional classification framework is proposed to survey and classify the main modeling approaches to integrate sequencing decisions in discrete time lotsizing models. Computational experiments are conducted to assess the models performance, in terms of running times and upper bounds, when solving real-word size instances. We also present a new commodity flow based formulation for the problem.

1 Introduction

In many production environments, switching between production runs of two different products triggers operations, such as machine adjustments and cleansing procedures. These setup operations, which are dependent on the sequence, consume scarce production time and may cause costs due to, for example, losses in raw materials or intermediate products. Consequently, the production sequence must be explicitly embedded in the lot definition. Lot sizing determines the level of production to satisfy deterministic product demand over the planning horizon. Sequencing establishes the order in which lots are executed within a time period, accounting for the sequence–dependent setup times and costs. Integration of these two problems enables the creation of better production plans than those obtained by solving the two problems hierarchically. Production plans are created with the objective of minimizing the overall costs consisting mainly of holding and setup, while satisfying the available capacity in each time period from which the expenditure in setup times is deducted. Examples of industries where these decisions must be taken concurrently are chemicals, drugs and pharmaceuticals, pulp and paper, textiles, foundries, glass container, and food and beverage, among many others.

The field of lotsizing and scheduling has received an increased attention from the research community due to its inherit applicability to real world problems as shown in the reviews by [6, 16, 9] and, recently, by the special issue [4]. Researchers have been incorporating additional scheduling decisions and features into lotsizing models to improve their realism and potential applicability. However, none of the aforementioned reviews focus on modeling techniques to integrate sequencing decisions into lotsizing models. Among the most important features when considering sequencing decisions in lotsizing models is to capture multiple production runs of the same product within a time period. Usually, setups obey the triangular inequality with respect to both the setup time and costs, i.e. it is more efficient to change directly between two products than via a third product, hence at most one setup for each product per time period is performed. Nevertheless, in some industries, contamination occurs when changing from one product to another implying additional cleansing operations. If a 'cleansing' or shortcut product can absorb contamination while being produced and thus replacing the cleansing operations, non-triangular setups appear. Therefore, models allowing for more than one production run of each product per time period potentially reduce setup times and costs.

Our contributions are as follows. We present a new classification framework to classify modeling approaches to lotsizing and scheduling with sequencing decisions. The new framework is used to survey and classify the different modeling approaches present in the literature. A new commodity flow based formulation to integrate sequencing decisions in discrete time lotsizing models is presented. Extensive computational results will allow a full evaluation of the pro and cons of the different modeling techniques.

2 Modeling sequence-dependent setups

Many models in the field of lotsizing and scheduling (LS) are expressed in the form of mixed integer programming (MIP) formulations. The advances observed in mathematical programming in the last years combined with the increase in computational power (hardware) and in the quality of general purpose mixed-integer programming commercial solvers (software) allowed standard lotsizing problems to be solved efficiently using exact methods. However, the development of tighter mathematical formulations is still mandatory to reduce the running times needed to solve real-world LS instances.

In this work we first survey and classify discrete time models for LS with sequencing decisions in two main dimensions: time resolution and scheduling technique (see Figure 1).

Most mathematical formulations for LS assume a planning horizon divided into a finite number of time buckets. Classifying LS models considering the resolution of the planning horizon has been commonly accepted in the community. According to this classification there are two main types of models: large and small bucket.

In large bucket models the planning horizon is partitioned into a small number of time periods, commonly representing a week or month. In each time period these models allow more than one setup. On the other hand, in small bucket models the planning horizon is divided into a large number of time periods, usually referred as micro-periods (e.g. days, hours or shifts). The assumption in these models is that at most one setup is performed per micro-period. The number of micro-periods may account for the maximum number of setup operations allowed in each time period. While in small bucket models the production sequence comes for free directly from the assumption of allowing at most one setup per micro-period, in large bucket models the use of decision variables similar to those of routing problem formulations requires sub-tour elimination constraints to correctly represent production sequences. The sub-tour elimination constraints are dramatically harder when allowing for multiple production runs in the same period, often resulting in an exponential number of constraints. Hence, large bucket models can be further divided into models allowing a single production run for each product per time period and those which consider multiple production runs.

The second dimension used for classification regards the technique used to capture sequencing decisions, which can be divided in two main approaches: product and sequence oriented formulations. In product oriented formulations sequences are defined explicitly by an MIP model, while in sequence oriented formulations the MIP model prescribes for each period or micro-period a sequence from a set of pre-determined sequences. Sequence oriented formulations for LS are easier to model and solve, but yield a major drawback coming from the fact that the number of decision variables grows exponentially with the number of products. On the other hand, product oriented formulations may have an exponential number of constraints.

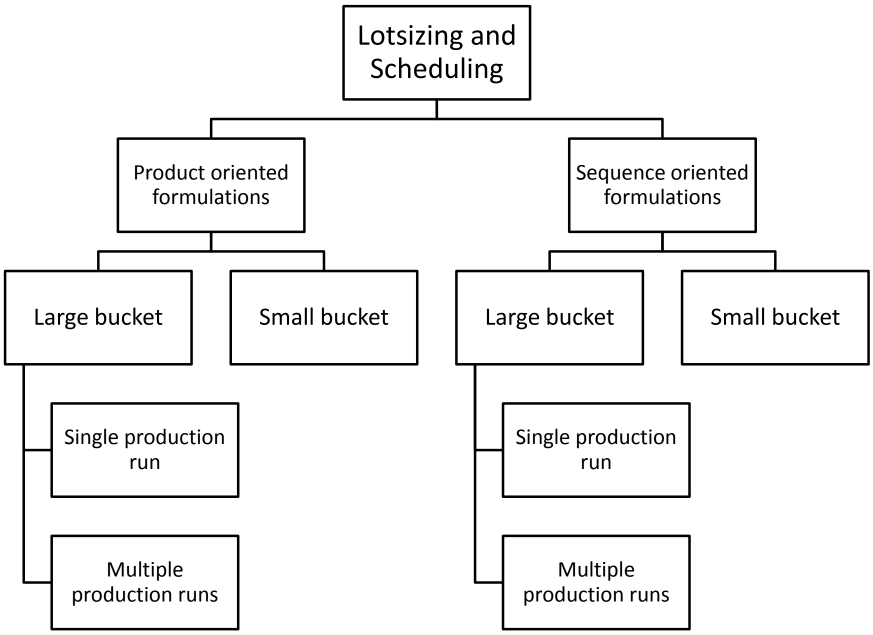

Figure 1: Proposed classification framework

Besides reviewing the models present in the literature we also introduce a new polynomial sized model formulation to LS which uses commodity flow based constraints to eliminate sub-tours and allows for multiple production runs of the same product.

The classification and the models reviewed are presented in Table 1.

The performance of the models reviewed in the proposed two-dimensional classification framework and of the new formulation is assessed by using a mixed-integer programming commercial solver. The goal is to test the capability of solving instances of real-world size. In addition, as many solution procedures for the LS combine heuristics with exact methods, we also assess their potential to be used in hybrid methods. Progressive interval heuristics are MIP-based heuristics which solve a series of partially relaxed MIP subproblems to construct an initial feasible solution to the original MIP and are widely used in the LS literature. Similarly, the 'exchange' (fix-and-optimize) improvement heuristic decomposes the set of integer variables in the original MIP to create MIP subproblems to re-optimize. The efficiency of these heuristics strongly depends on the efficient MIP formulations.

Table 1: Classification of lotsizing and scheduling models with sequencing decisions

			Scheduling Technique			
			Product Oriented Formulations		Sequence Oriented Formulations	
Time Resolution	Large Bucket	Single Production Run	Time dependent	Smith-Daniels and Ritzman [14]	subproblems - TSP	Haase and Kimms [8]
Time Resolution	Large Bucket	Single Production Run	Miller-Tucker-Zemlin based	Haase [7]		
Time Resolution	Large Bucket	Single Production Run	Miller-Tucker-Zemlin based with alpha sub-tours	Almada-Lobo et al. [2]		
Time Resolution	Large Bucket	Single Production Run	Exponential sized	Almada-Lobo et al. [2]		
Time Resolution	Large Bucket	Multiple Production Runs	Exponential sized	Belvaux and Wolsey [3]	subproblems - Prize collecting ATSP	Guimarães et al. [1]
Time Resolution	Large Bucket	Multiple Production Runs	Exponential sized	Menezes et al. [11]		
Time Resolution	Large Bucket	Multiple Production Runs	Multi-commodity flow	Sarin et al. [13]		
Time Resolution	Large Bucket	Multiple Production Runs	Single-commodity flow	this work		
Time Resolution	Small Bucket		Pre-definied maximum number of setups (GLSP)	Meyr [12]	Split Sequences	Kang et al. [10]
Time Resolution	Small Bucket		Network flow reformulation of changeovers	Wolsey [15]		
Time Resolution	Small Bucket		Pre-definied maximum number of setups	Clark and Clark [5]		

During the computational experiments we analyze the trade-offs present in these different modeling approaches. First, we study the correlation between the additional complexity introduced by allowing multiple production runs of the same product and the solution quality obtained when a time limit is imposed to the solution. Second, we compare the use of exponential number of constraints and variables against the use of compact model formulations.

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Complexity results for the single item uncapacitated lot-sizing problem with time-dependent batch sizes

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Abstract
We consider the uncapacitated lot sizing problem with batch delivery. We study the complexity of this problem in the case of time-dependent batch sizes. We prove four special cases to be NP-hard and two other to be polynomially solvable. As a consequence of our results, if any one of the cost parameters is allowed to be time-dependent, the problem becomes NP-hard.

1 Introduction

We consider the single item uncapacitated lot-sizing problem where the ordered quantities are delivered by batch (*i.e.* truck or container) from an external supplier. Each batch ordered incurs a fixed cost, independently of the number of units actually in the batch. The same problem can be seen as a single machine with an unlimited capacity, producing a single item per batches of certain size. We call this problem UBLS for uncapacitated batch lot-sizing. As in the classical lot sizing problem, a setup cost f_t is paid for a period of positive production, a procurement cost p_t per unit ordered, and a holding cost h_t per unit of product in stock between period t and $t+1$. Demands d are known over T periods and are to be satisfied without backlogging.

For the batch delivery, we adopt an FTL (Full Truck Load) cost structure, where, in addition to the classical cost structure, a fixed cost k_t is paid for each batch used in period t. This cost structure is also referred to as stepwise costs in the literature. Notice that our model allows to order incomplete batches. We denote by B_t the size of the batch in period t. The overall procurement cost $q_t(x)$ for an amount x of products ordered in period t is thus given by :

$$q(0) = 0 \text{ and } q_t(x) = f_t + p_t x + \lceil x/B_t \rceil k_t \text{ for } x > 0$$

A formulation of the problem is given below, where x_t represents the amount of products ordered in period t and s_t the stock level at the end of period t. Without loss of generality we assume no initial inventory, that is $s_0 = 0$. Notice that this formulation is non-linear due to the procurement cost $q_t(x)$.

$$(UBLS) \begin{cases} \min & \sum_{t=1}^{T} (q_t(x_t) + h_t s_t) \\ s.t. & s_{t-1} + x_t = d_t + s_t \quad \forall t = 1, \dots, T \\ & x_t, s_t \in \mathbb{R}_+ \quad \forall t = 1, \dots, T \end{cases}$$

For ease of the reading, in the remaining the notation $(f_t/k_t/h_t/p_t)$ is used to designate the assumptions adopted on the cost parameters (setup cost/fixed cost per batch/unit holding cost/unit procurement cost). The field for a parameter α will take either the value '$-$' if α is null, 'α' if it is assumed stationary and finally 'α_t' if it is allowed to be time-dependent. For example, the rightmost problem in Figure 1 is designated by $(f_t/k/-/-)$ in our notation, corresponding to UBLS instances with time-dependent setup costs, stationary fixed cost per batch and no unit procurement nor holding costs.

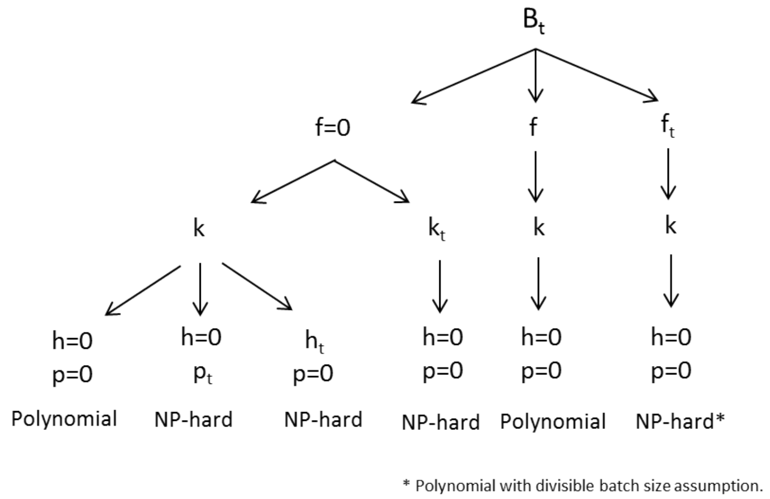

Figure 1: Complexity of different special cases of UBLS.

The UBLS problem is well-studied in the literature, but as far as we know, all the papers restrict to the case of a stationary batch size. To the best of our knewledge, the first study considering a stepwise cost (but without setup cost) is due to Lippman [4]. The author proposes an $\mathcal{O}(T^5)$ time algorithm for UBLS with $(-/k_t/h_t/p_t)$ and without backlogging. For the same problem, Pochet and Wolsey [5] improve this result to an $\mathcal{O}(T^2 \min(T, B))$ time algorithm. Lee [2] studies a similar structure, with a non null setup cost but assuming stationary cost parameters: $(f/k/h/p)$ and proposes an $\mathcal{O}(T^4)$ time algorithm. Li *et al.* [3] consider the more general problem both with time-dependent cost parameters $(f_t/k_t/h_t/p_t)$ and with backlogging, and propose an $\mathcal{O}(T^3)$ time algorithm. The capacitated lot-sizing problem with batch production has also been studied in the literature. See two recent studies, van Vyve [6] and Akbalik and Rapine [1] for the polynomial time algorithms proposed.

As a conclusion, when restricted to stationary batch sizes, stepwise costs appear to do not alter the complexity status of the lot-sizing problem. That is, both the uncapacitated and capacitated lot-sizing problems remain polynomial, even when backlog is allowed. In contrast, we establish that UBLS with time-dependent batch sizes is $\mathcal{NP}$-hard, even if all but one cost parameters are stationary. Figure 1 gives a synthetic representation of our results.

General case : $(f_t/k_t/h_t/p_t)$

For the most general case of UBLS with all cost parameters being time-dependent, to our knowledge no general dominance properties exist in the literature, but a general pseudo-polynomial time dynamic programming algorithm can be adapted to solve it. Note that UBLS can be solved in polynomial time if the fixed cost per batch is null ($k_t = 0$), all other cost parameters remaining time-dependent. In this case UBLS is equivalent to the classical uncapacitated lot sizing problem.

Case $(-/k_t/-/-)$

We show that UBLS is $\mathcal{NP}$-hard when both fixed cost per batch and batch sizes are time-dependent, even with no setup cost, no unit procurement cost and no unit holding cost. The reduction is immediate from the UNBOUNDED KNAPSACK PROBLEM.

Cases $(-/k/p_t/-)$ and $(-/k/-/h_t)$

UBLS with time-dependent unit procurement cost p_t and time-dependent batch size is shown to be $\mathcal{NP}$-hard with all other cost parameters stationary. It remains NP-hard even if the setup and the holding costs are null. We made the reduction from the MONEY CHANGE PROBLEM. Using the classical result that any instance of the lot-sizing problem can be transformed into an equivalent instance with no procurement costs and modified unit holding costs, we can also show the NP-hardness of the second case with time-dependent unit holding cost h_t.

Case $(-/k/-/-)$

The only cost is a stationary fixed cost k per batch. It is immediate to see that an optimal policy produces only full batches. A greedy approach selecting at each step the largest batch size period solves the problem in $\mathcal{O}(T)$ time.

Case $(f_t/k/-/-)$

We consider the setup costs f as the only time-dependent cost parameters. This means that once the setup paid, any batch in any period has the same cost k. Although this problem seems simple, it happens to be NP-hard. A more efficient pseudo-polynomial time algorithm is given for this case, in $\mathcal{O}(T \sum_t B_t)$. We also show that the problem becomes polynomially solvable for *divisible* batch sizes, in $\mathcal{O}(T^3 \log T)$ time.

Case $(f/k/-/-)$

We finally consider the case where only the batch sizes are time-dependent and all the cost parameters are stationary. We restrict our attention to a null unit holding cost, that is $h = 0$. We show that this case can be solved in time $\mathcal{O}(T^3)$.

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Lot-sizing with minimum batch sizes

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Abstract
We study lot-sizing problems with batches and a minimum order quantity. Two variants are considered: with and without a capacity constraint on the number of batches per period. We develop dynamic programs for both cases and see that these can be simplified significantly if the costs have a special structure that is similar to Wagner-Whitin (non-speculative) costs.

1 Introduction

Often, production takes place in several runs (batches) of a certain (maximum) size. Batches can also have a minimum size, for instance because of technical restrictions on a machine or because of supplier restrictions. This problem is also related to carbon emission reduction. Today, just-in-time production is a very popular production strategy. However, it often leads to carbon emission levels that are far from optimal, because of its frequent less-than-truckload shipments and/or frequent change-overs on machines. By imposing a minimum order quantity in each period, we prevent products from being transported by almost empty vehicles or machines from producing only very few units of a product per batch.

There may also be a capacity constraint on the total number of batches that may be produced within a period. We consider both this capacitated variant and an uncapacitated variant.

[1] get lot-sizing with minimum batch sizes as a special case of their problem, which extends work in [2]. Lot-sizing problems with batches but without a minimum order quantity (i.e. with minimum order quantity zero) are studied by [3], in which also a good overview of related literature can be found. Lot-sizing problems with a minimum order quantity but without batches are studied by [4], [5] and [6].

2 Problem definition

Sets and parameters

Let T be the number of time periods and $\mathcal{T} = \{1, \ldots, T\}$ the set of all periods. The demand in period t is denoted by d_t. Let $K_t(y_t)$ denote the fixed costs per batch if y_t

batches are produced in period t. $K_t(y_t) := \sum_{i=1}^{y_t} K_t^i$ and K_t^i are the fixed costs of the ith batch in period t, where $K_t^i \geq K_t^{i+1} \forall i \in \mathbb{N}, t \in \mathcal{T}$. That means that the fixed costs may differ per batch, as long as they are decreasing in the number of batches. Let L and F be the minimum (Lowest) and maximum (Full) production quantity in one batch, respectively. We assume that these are constant over batches and time. The production capacity in each period is denoted by C. We will assume that this production quantity is also constant over time and a multiple of F, the maximum batch size, i.e. $\exists k \in \mathbb{N} : C = kF$.

Let $p_t(x_t)$ and $h_t(I_t)$ be the production, respectively holding cost function in period t; they are assumed to be nonnegative functions and $h_t(I_t)$ should be concave. $p_t(x_t)$ should be concave 'within' the interval of allowed production quantities for each batch. That is, in each interval $[Lk, Fk]$ for $k \leq \tilde{k}$ and $(F(k-1), Fk]$ for $k > \tilde{k}$ ($k \in \mathbb{N}$), where $\tilde{k} := \max\{k \in \mathbb{N} : kL > (k-1)F\}$.

Variables

Let x_t be the production quantity in period t, for all $t \in \mathcal{T}$. y_t is the number of batches produced in period t. I_t is the inventory at the end of period t.

Model

$$\min \sum_{t \in \mathcal{T}} \left(K_t(y_t) + p_t(x_t) + h_t(I_t)\right) \tag{1}$$

$$\begin{aligned} \text{s.t.} \quad I_{t-1} + x_t &= I_t + d_t && t \in \mathcal{T} && (2)\\ Ly_t \leq x_t &\leq Fy_t && t \in \mathcal{T} && (3)\\ x_t &\leq C && t \in \mathcal{T} && (4)\\ I_0 &= 0 && && (5)\\ x_t, I_t &\geq 0 && t \in \mathcal{T} && (6)\\ y_t &\in \mathbb{N} && t \in \mathcal{T} && (7) \end{aligned}$$

In the uncapacitated variant, constraint (4) is omitted.

3 Algorithms

First, we derive several structural properties of an optimal solution. Based on these properties, we develop dynamic programming algorithms to solve the problems described above.

We find that the uncapacitated variant can be solved in $\mathcal{O}\left(T^5\left(\min\left\{\frac{TF}{F-L}, \frac{D_{1,T}}{L}\right\}\right)^2\right) \subseteq \mathcal{O}\left(T^7\left(\frac{F}{F-L}\right)^2\right)$. This reduces to $\mathcal{O}\left(T^4\left(\min\left\{\frac{TF}{F-L}, \frac{D_{1,T}}{L}\right\}\right)^2\right) \subseteq \mathcal{O}\left(T^6\left(\frac{F}{F-L}\right)^2\right)$ if the production cost functions are fixed-plus-linear for each batch.

These running times are polynomial for general K_t^i, that is, if different batches have 'completely different' costs. If the function K_t can be represented in a more compact way (i.e. fully polynomial in the number of batches), then the dynamic program is pseudo-polynomial in the input size. However, the algorithm still runs in polynomial time for a fixed ratio of $\frac{F}{F-L}$. This includes the case where L divides F (see [1]), because in that case $\frac{F}{F-L} \leq 2$.

We study the following special case in more detail. Let the (production and holding) costs be linear and non-speculative (Wagner-Whitin). Let the set-up costs K_t^i be equal for each batch i within period t except for the first one (K_t^1), which may be larger, i.e., $K_t^i = K_t^j \forall i, j \geq 2$ and $K_t^1 \geq K_t^2$, and let the set-up costs for batches 2 and higher be non-increasing over time, i.e., $K_t^i \geq K_s^i \forall t < s, i \geq 2$. In this case, an optimal solution has several additional properties, so that the complexity of the dynamic program for the uncapacitated variant can be reduced to $\mathcal{O}\left(T^4\right)$. Moreover, in this special case, a dynamic program can be constructed for the capacitated variant that runs in $\mathcal{O}\left(T^5\right)$ time.

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Dynamic capacitated lot-sizing with parallel common setup operators

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Abstract
We observed a capacitated lot-sizing problem with linked lot-sizes, multiple machines and sequence-dependent setup costs and times in a German food company. Setup operations are carried out by parallel common setup operators. As a common setup operator can perform only one setup at once, setup operations that are carried out by a certain operator must be synchronized to avoid intersections. We present a model formulation as well as a heuristic solution approach for this lot-sizing problem .

1 Introduction

Classical lot-sizing problems are primarily concerned with the question about when and how much production of a certain product occurs considering setup and inventory holding costs. However, in a German factory producing food, as well as in several automotive companies, we observed that sometimes specialized resources are required to carry out setup operations. Since these resources are not able to handle more than one setup operation at the same time, they have to be included into the decision problem to avoid overlapping setups.

Models, which consider one common setup operator to synchronize setup operations, were proposed by [1] and [2]. While [1] use a PLSP-based model formulation, [2] extend the Capacitated Lot Sizing Problem with linked lot-sizes, multiple machines and sequence-dependent setups (CLSP-L-MM-SD) by [3].

Nevertheless, the consideration of only one common setup operator does not entirely represent the planning situation in the mentioned factory, as there are more (parallel) common setup operators. Therefore, we present a model that extends the CLSP-L-MM-SD with one common setup operator (CLSP-L-MM-SD-CSR) by parallel common setup operators (CLSP-L-MM-SD-PCSR) in order to get a precise mathematical formulation for this practical case. Due to long computation times for the exact solution, we additionally present a heuristic solution approach.

2 Problem description

In the considered practical case, products with dynamic demands are produced on multiple capacitated resources. Setup times and costs are sequence-dependent and the setup state of a production resource can be carried over to the next periods. Each production resource requires a setup operator which can handle only one production resource at a time. Therefore, setup operations on all machines using a certain setup operator have to be coordinated. A production resource can use any available setup operator. Intersections between setup operations are allowed, if they are carried out by different setup operators. In detail, the underlying problem has the following characteristics:

- The planning horizon is divided into T periods.
- K products are produced.
- An external demand d_{kt} for product k in period t must be met.
- Backlogging and overtime are not allowed.
- M production resources (machines) with period-specified capacities b_t^m are available.
- Each product is uniquely assigned to a single machine ($k \in \mathcal{K}_m$, $m = 1, 2, \ldots, M$).
- A production lot is always associated with a setup operation performed by one of the setup resources.
- Each setup operation is linked with sequence-dependent setup-times and -costs.
- Setup states are carried over to the next period, so production of a lot may run over several consecutive periods.
- The objective is to minimize the sum of holding and setup costs.

3 Solution

We develop a model formulation using the big-bucket model CLSP-L-MM-SD-CSR by [2] and the Simple Plant Location reformulation. The CLSP-L-MM-SD-PCSRspl is very complex due to a high number of binary setup variables generated through sequence-dependence and the coordination of setup operations. Thus, we also develop a Fix-and-Optimize heuristic based on the idea of [4] with a resource/time-oriented decomposition to solve large problem instances. The model formulations as well as the heuristic are implemented with OPL and solved with CPLEX 12.2.

Problem instances comprise 50 products, 7 machines, 2 setup operators and 2 up to 12 periods. The average capacity utilization is 66%, at which it respectively amounts 90% for machine 5 and 7. Table 1 shows the numerical results for the exact solution approach on an Intel(R) Xeon(R), 2.67GHz Core Duo with 3.5 GB RAM and Windows 7 (32-bit). For solving a problem instance a time limit of 3000 seconds was set.

Periods	LB	UB	Gap	Seconds
2	370.33	370.33	0.00%	6
3	673.17	673.17	0.00%	78
4	740.90	741.14	0.03 %	3000
5	780.09	780.65	0.07%	3000
6	881.69	894.21	1.37 %	3000
7	944.93	1026.45	7.94 %	3000
8	995.89	1190.34	16.34 %	3000
9-12	n/a	n/a	n/a	memory overflow

Table 1: Numerical results for the CLSP-L-MM-SD-PCSRspl

The results show that only small problem instances can be solved exactly with CPLEX, whereas no feasible solution can be found for problems with 9 or more periods. For comparison reasons, Table 2 presents computation times for the Fix-and-Optimize heuristic and deviations of the objective function values.

Periods	Seconds	Deviation
2	16	-7.03%
3	38	-4.63%
4	210	-7.22%
5	217	-9.45%
6	415	-4.25%
7	608	-1.24%
8	744	+0.35%
9	869	n/a
10	1405	n/a
11	1590	n/a
12	1796	n/a

Table 2: Numerical results for the Fix-and-Optimize heuristic

To get comparable objective function values, the solution time of the Fix-and-Optimize heuristic is the time limit for the exact solution. In Table 2, the resulting deviation of the objective values can be seen. For the data sets with 7 and 8 periods, the given time limit is not sufficient to get a feasible solution. Therefore, the first feasible solution is taken to calculate the deviation.

In comparison to the exact solution approach, the heuristic is able to generate feasible solutions even for large problems. Moreover, Table 2 shows that the heuristic can not only generate a feasible solution in less time (problems with 7 or more periods) but also provides a better objective function value for the considered problem with 8 periods.

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First Results on Multi-Level Capacitated Lot-Sizing in Closed-Loop Supply Chains

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Abstract
In this paper, we present a new model formulation for a multi-level capacitated lot-sizing problem with returns and remanufacturing (MLCLSP-RM) in closed loop supply chains. After disassembling returned goods, components can be remanufactured or newly produced. However, the production times and costs of the returned components depend on the quality level. Whenever a product or component is (re)manufactured the respective machine has to be set up, which causes setup costs and/or setup times. The objective of the MLCLSP-RM is to determine a feasible production schedule which minimizes the sum of setup, (re)manufacturing, disposing, holding, and overtime costs. For the solution of the MLCLSP-RM, we adapt known MIP-based solution approaches from literature, e.g., the Fix-and-Optimize heuristic by [2]. The analysis of first numerical results shows the high solution quality of the proposed heuristics.

1 Introduction

In closed-loop supply chains, in addition to a forward-oriented supply chain from the manufacturer to the customer, the reverse direction also has to be taken into account. In this setting, customers return old and used products to the manufacturer at the product's end-of-use. The manufacturer can decide whether to remanufacture or dispose these returned products. In case of remanufacturing, the product returns are firstly disassembled and after remanufacturing these returned components are as good as new components.

2 Problem Statement and Model Formulation

We assume that K products are (re)manufactured in a multi-level production environment on a production system containing M different capacitated machines. Each resource can only manufacture or remanufacture products, but not both. (Re)manufacturing a product leads to setup times and costs in each period the respective machine is set up for the respective product. The primary demand is known and has to be fulfilled completely in the respective period. As the returned products are disassembled in advance, the quantities of returned components and the respective quality level are known in each period. The (re)manufacturing time and costs per unit are given and independent of the production quantity. However, they depend on the quality level of the returned components. Furthermore, the returned components can be disposed causing disposing costs. The holding costs are proportional to the inventory at the end of a period. The objective of the MLCLSP-RM is to generate a production plan which minimizes setup, (re)manufacturing, disposing, holding, and overtime costs over the whole planning horizon.

Table 1: Notation used for the MLCLSP-RM

Indices and index sets:	
$\mathcal{K}$	set of products ($k \in \{1, \ldots, K\}$)
$\mathcal{L}$	set of quality levels ($l \in \{1, \ldots, L\}$)
$\mathcal{M}$	set of resources ($m \in \{1, \ldots, M\}$)
$\mathcal{T}$	set of periods ($t \in \{1, \ldots, T\}$)
Subsets:	
$\mathcal{K}_m$	set of products requiring machine m
$\mathcal{M}^r$	set of resources for remanufacturing
$\mathcal{M}^s$	set of resources for manufacturing
$\mathcal{N}_k$	set of immediate successors of product k
Parameters:	
a_{ki}	number of units of product k required to produce one unit of product i
c_{mt}	available capacity of the resource m in period t
dc_{kl}	disposing cost of returned product k with quality level l
hc_k	holding cost of product k
hc^r_{kl}	holding cost for returns of product k with quality level l
M_{kt}	big number for product k in period t
oc_m	overtime cost at machine m
pc_k	production cost of product k
pc^r_{kl}	remanufacturing cost of product k with quality level l
pd_{kt}	primary demand of product k in period t
r_{ktl}	returns of product k with quality level l in period t
sc_k	setup cost of product k
sc^r_k	setup cost of returned product k
tp_k	production time of product k
tp^r_{kl}	remanufacturing time of product k with quality level l
ts_k	setup time of product k
ts^r_k	setup time of returned product k
Decision variables:	
D_{ktl}	disposal quantity of product k with quality level l in period t
O_{mt}	amount of overtime at machine m in period t
Q_{kt}	production quantity of product k in period t
Q^r_{ktl}	remanufacturing quantity of product k with quality level l in period t
Y_{kt}	inventory of product k at the end of period t
Y^r_{ktl}	inventory of returned product k with quality level l at the end of period t
γ_{kt}	binary setup variable of product k in period t
γ^r_{kt}	binary setup variable for remanufacturing product k in period t

Following [3], the MLCLSP-RM can be stated using the notation in Table 1:

$$\min Z = \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\left(hc_k \cdot Y_{kt} + \sum_{l\in\mathcal{L}} hc^r_{kl} \cdot Y^r_{ktl}\right) + \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\left(pc_k \cdot Q_{kt} + \sum_{l\in\mathcal{L}} pc^r_{kl} \cdot Q^r_{ktl}\right) \tag{1}$$

$$+ \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}(sc_k \cdot \gamma_{kt} + sc^r_k \cdot \gamma^r_{kt}) + \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{l\in\mathcal{L}} dc_{kl} \cdot D_{ktl} + \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} oc_m \cdot O_{mt}$$

subject to

$$Y_{k,t-1} + Q_{kt} + \sum_{l\in\mathcal{L}} Q^r_{ktl} - \sum_{i\in\mathcal{N}_k} a_{ki} \cdot \left(Q_{it} + \sum_{l\in\mathcal{L}} Q^r_{itl}\right) = pd_{kt} + Y_{kt} \qquad \forall\, k,t \tag{2}$$

$$Y^r_{k,t-1,l} + r_{ktl} = Q^r_{ktl} + D_{ktl} + Y^r_{ktl} \qquad \forall\, k,t,l \tag{3}$$

$$\sum_{k\in\mathcal{K}_m} (tp_k \cdot Q_{kt} + ts_k \cdot \gamma_{kt}) \le c_{mt} + O_{mt} \qquad \forall\, m \in \mathcal{M}^s, t \tag{4}$$

$$\sum_{k\in\mathcal{K}_m} \left(\sum_{l\in\mathcal{L}} tp^r_{kl} \cdot Q^r_{ktl} + ts^r_k \cdot \gamma^r_{kt}\right) \le c_{mt} + O_{mt} \qquad \forall\, m \in \mathcal{M}^r, t \tag{5}$$

$$Q_{kt} \le M_{kt} \cdot \gamma_{kt} \qquad \forall\, k,t \tag{6}$$

$$\sum_{l\in\mathcal{L}} Q^r_{ktl} \le M_{kt} \cdot \gamma^r_{kt} \qquad \forall\, k,t \tag{7}$$

The objective function (1) minimizes the sum of inventory holding, production, remanufacturing, setup, disposing and overtime costs. Equations (2) and (3) represent the inventory balance constraints for manufacturing or remanufacturing, respectively. Inequalities (4) and (5) ensure that (re)manufacturing quantities and setups meet the capacity constraints in each period. Inequalities (6) and (7) link the variables of (re)manufacturing quantities to the setup variables, i. e., if product k is produced in period t, the respective machine has to be set up for this product in the respective period.

3 Outline of the Solution Approach

For solving the MLCLSP-RM, we adapt, e.g., the Fix-and-Optimize heuristic (cf. [2]). The idea of the Fix-and-Optimize heuristic is to solve a sequence of subproblems in an iterative fashion. In each subproblem, the number of "free" binary setup variables is limited as most of the binary setup variables are fixed to a constant setup state. this subproblem can be solved to optimality quickly. This (optimal) solution describes a new temporary solution for the binary setup variables. At least some of them are fixed in the next subproblem when a new subset of binary variables is solved to optimality. In contrast, the real-valued decision variables are never fixed for all products, periods and machines.

A first numerical study has shown that the Fix-and-Optimize heuristic provides high quality results for the MLCLSP-RM with respect to both solution time and solution quality.

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Linear programming models for the stochastic dynamic capacitated lot sizing problem

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Abstract

The non-linear optimization problem of the stochastic dynamic capacitated lot sizing problem can be reformulated as a solvable linear program by approximating the non linear functions of backlog and inventory with a series of piecewise linear segments. Since the fill rate criterion is very popular in industrial practice, a model formulation with β service-level constraint is presented. It is assumed that, according to the static-uncertainty strategy of [1], all decisions concerning the time and the production quantities are made in advance for the entire planning horizon regardless of the realization of the demands.

1 Introduction

The observed problem is to determine production quantities to satisfy time-varying random period demands over a finite discrete time horizon so as to minimize the sum of setup and holding costs under consideration of a fill-rate constraint. Especially, in a planning environment with uncertain demand the service-level towards the customers is at least as important as cost efficiency. On the one hand in industrial practice many manufacturers have to face demand uncertainty and on the other they have to guarantee their customers a predefined service-level. Anyway, conventional production planning systems neglect demand uncertainty and use deterministic planning approaches. Therefore, the stochastic dynamic capacitated lot sizing problem is presented.

It is assumed that according to the static-uncertainty strategy of [1], all decisions concerning the time and the production quantities are made in advance for the entire planning horizon, which is equivalent to use a frozen schedule. We assume that for every period t and every product k the demand is a random variable D_{kt} ($t = 1, 2, \ldots, T;\ k = 1, 2, \ldots, K$). Demand that cannot be filled immediately from stock on hand is backordered. As the precise quantification of shortage penalty costs which involve intangible factors such as loss of customer goodwill is very difficult to quantify, usually technical performance measures are applied in practice. Since the fill rate criterion is very popular in industrial practice, a model formulation with a finite horizon β_t service-level and cyclic β_{cyc} service-level[1][2] constraint is presented.

[1] Cf. [4] p. 36.
[2] Cf. [6] p. 5183.

2 Modeling Approach for the Stochastic Dynamic Lot-sizing Problem

Assuming independent and normal distributed period demand the random variables of the expected backlog and the expected inventory are convex functions of the cumulated production quantity. [3] propose to replace the non-linear functions of expected physical inventory $E\left\{I_{kt}^{p}\right\}$ and backlog $E\left\{I_{kt}^{f}\right\}$ to approximate the stochastic multi-item capacitated lot-sizing problem (SMICLSP) with δ service-level. This linearization technique can be used to approximate the SMICLSP with β_t and β_{cyc} service-level constraint.

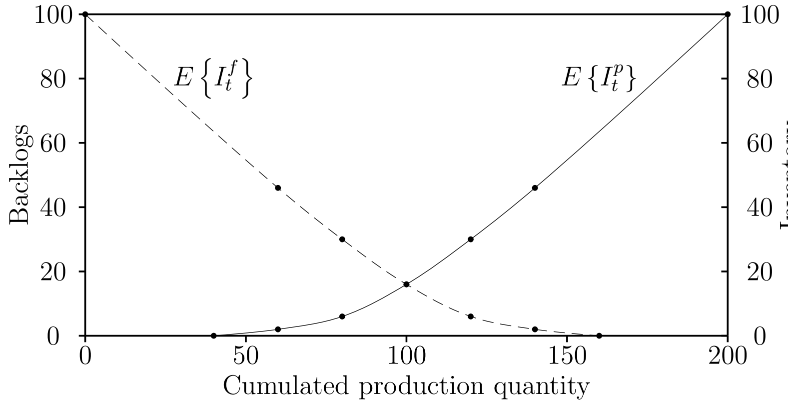

Figure 1: Approximation of expected backlogs and physical inventory

The idea of the piecewise linear approximation model is to replace the non-linear functions by piecewise linear segments so that the problem can be converted into a solvable linear model. The quality of the results of the approximation model depends on appropriate chosen knots, the points at which the values of a function are assigned. The functions of the expected backlog and the expected physical inventory are illustrated in Figure 1. These points have to be concentrated in the area where the convexity of the curve is strongest. Following the assumptions of the normal distribution, 99.73% of the observations fall within the interval $\mu \pm 3\sigma$[3]. [3] showed that such a linearization is required for each period t and each product k for the expected inventory and the expected backlog as functions of the cumulated production up to period t[4]. For a model formulation with fill-rate constraint it is additionally required to determine the period specific backorders. Therefore, it is necessary to compute the supporting points for the approximation via the first-order loss function.

3 First Numerical Results

To test the quality of the proposed optimization model a numerical experiment is conducted which is based on problem instances presented by [5]. In total, 90 problem in-

[3]See table of standard normal distribution.

[4]Cf. [3] pp. 12–15.

stances were generated. The costs of the presented solution procedure that combines column generation (CGN) and the ABC_β Heuristic is compared to the ABC_β Heuristic presented by [6]. The numerical experiment was performed for a SMICLSP with β_{cyc} service-level and solved with a Fix & Optimize Heuristic[5]. The results will be served as reference for the linearized optimization model. The periods demands are assumed to be normally distributed and the parameters are shown in Table 1.

Table 1: Parameters for the numerical experiment

Number of products	10
Number of periods	10
Capacity	1.1 · average demand
Fill-rate β_{cyc}	0.80,0.90,0.98
Mean period demand	Continuous uniform U(0,100)
Coefficient of variation	0.15, 0.20, 0.25, 0.30, 0.35
Time-between-orders, TBO	Discrete uniform U(1,5,10)
Setup Times	0

The computations are based on up to 200 linear segments, the accepted relative MIP-Gap $\Delta_{MIP} = 2\%$ and a time-limit for each subproblem of 120 seconds. The computational tests were performed on an standard PC (Windows 7 -32 bit-, 4 * 2.83 GHz, 3.5 GB RAM) and the problem instances were solved with CPLEX (12.2). The results of the numerical experiments for data sets with high capacity utilization are presented in Table 2. The results of each setting are based up on 10 data sets. The relative cost savings against the Column Generation Heuristic are presented in column Δ costs CGN and the savings against the ABC_β Heuristic are shown in column Δ costs ABC_β.

Table 2: Results for low capacities

β	TBO	Δ costs CGN	Δ costs ABC_β
0.80	01	4.90%	13.38%
0.80	05	4.33%	15.22%
0.80	10	8.28%	12.72%
0.90	01	5.06%	11.50%
0.90	05	15.57%	23.46%
0.90	10	20.00%	21.74%
0.98	01	n/a	n/a
0.98	05	n/a	n/a
0.98	10	n/a	n/a
	ø	9.29%	16.34%

The numerical results in Table 2 have a capacity utilization ≥ 90%. In practical planning situations capacities are generally scarce and therefore the results underline the performance of the linearization technique for practical applications. It is remarkable that the CGN Heuristic gives in just 31 of 60 experiments results, whereby the linearized optimization model is able to solve all data sets. In case of $\beta_{cyc} = 0.98$ no feasible solution could be found by the CGN Heuristic and the ABC_β Heuristic. The Fix & Optimize heuristic gives solutions with overage time usage. Furthermore, the linearized model gives in 28 of 31 experiments the best solution. The ABC_β Heuristic achieves in 36 of

[5]Cf. [2].

60 a result, but was outperformed by the linearized optimization model in all cases. On average the solution quality of the Fix & Optimize Heuristic is 9.29% better than the CGN Heuristic and 16.34% better than the ABC_β Heuristic.

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Static-Dynamic Uncertainty Strategy for a Single-Item Stochastic Inventory Control Problem

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Abstract
We consider a single-stage inventory system facing non-stationary stochastic demand of the customers in a finite planning horizon. Motivated by the practice, the replenishment times need to be determined and frozen once and for all at the beginning of the horizon while decision on the exact replenishment quantities can be deferred until the replenishment time. This operating scheme is refereed to as "static-dynamic uncertainty" strategy in the literature. We consider dynamic fixed-ordering and linear end-of-period holding and penalty costs. We prove that the optimal ordering policy is a base stock policy. Since an exponential exhaustive search based on dynamic programming yields the optimal ordering periods and the associated base stock levels, it is not possible to compute the optimal policy parameters for longer planning horizons. Thus, we develop two heuristics. Numerical experiments show that both heuristics perform well in terms of solution quality and scale-up efficiently, hence, any practically relevant large instance can be solved in reasonable time.

1 Introduction and contributions

In inventory planning, freezing schedules for the timing of deliveries in advance, taking account of future uncertainties, is of practical interest. This need to fix the deliveries in advance, whilst allowing reasonable flexibility in the order size, has been at the heart of many industrial problems such as coordinating suppliers and buyers in supply chain partnerships [4]; managing joint replenishments for multiple items [8] and planning for shipment consolidation in logistics [6]; master planning and leveling workload in Advanced Planning Systems [8]; buying raw materials on fluctuating price markets in purchasing [5]; etc.

Freezing delivery schedules alleviate the supplier-buyer coordination in supply chains susceptible to system nervousness, which arises when a formerly fixed order request for a certain period is replanned later. The deviations causing nervousness may be in the form of quantity adjustments and/or changes in delivery requests. Inderfurth [4] notes that nervousness due to deviations in delivery requests is considered as the most serious in practice and is referred to as setup instability. Blackburn et al. [1] encourage deliveries in periods where they are scheduled previously for dealing with the problem of nervousness.

Silver et al. [8, pp.236-237] point out that freezing schedules is particularly appealing when items are ordered from the same supplier or require resource sharing. In such a case, all items in a coordinated group can be given the same replenishment period. This also allows a reasonable prediction of the level of the workload on the staff involved and is particularly suitable for advanced planning environments.

Shipment consolidation in logistics management is another area that benefits from scheduled deliveries. Mutlu et al. [6] describe shipment consolidation as the "practice of combining small size shipments into a larger load with the aim of benefiting from scale economies associated with transportation costs". The shipment consolidation policy using scheduled deliveries is called the "time-based policy" and noted to be popular in practice. In this policy, arriving orders are combined to form a large load, and consolidated shipments are released at periodic intervals. It is emphasized that this policy is important in terms of delivery reliability since it allows logistics providers to quote a delivery time.

In this paper, we consider the inventory problem faced by a manager who replenishes stock using the following practice. At the beginning of the planning horizon, he decides on the number and exact timing of delivery requests once and for all. This decision constitutes the "static" part of the manager's "static-dynamic uncertainty" strategy, name coined by Bookbinder and Tan [3]. Each delivery request incurs a fixed cost. The exact order quantities for deliveries are determined only after observing the realized demands until that time. This decision constitutes the "dynamic" part of the strategy. The demand process in this paper is assumed to be the only source of uncertainty and follow a non-stationary pattern over a finite planning horizon.

The static-dynamic uncertainty strategy is described first in [3]. They assume non-stationary demand and suggest a two-step heuristic solution method. In the first step, future replenishment periods are fixed at the beginning of the planning horizon using a Wagner-Whitin type model. In the second step, subsequent order quantities are determined on the basis of demands that have become known at a later point in time. Proportional end-of-period inventory holding and fixed-ordering costs are taken into account. Instead of adopting a penalty cost approach, service level constraints are imposed in each period. In a related work, Bookbinder and Tan [2] introduced and tested a rolling horizon framework. Tarim and Kingsman [10] address the same problem and provide a mixed integer programming (MIP) model to simultaneously answer the questions on the exact timing of future replenishments and corresponding order quantities. Tarim et al. [9] mainly focus on the computational issues and provide an efficient computation approach to solve the MIP model in [10]. Tempelmeier [12] addresses the same problem under a fill rate constraint as a service measure. Finally, Tarim and Kingsman [11] relax the service level constraints and present an approximate model for the penalty cost case with normally distributed demands. In all the aforementioned works the inventory control policy is in the form of static-dynamic uncertainty strategy. Although the demand process is assumed to be non-stationary and stochastic, all these papers formulate certainty equivalent mathematical programs and analyze the resulting deterministic problems.

Our contribution in this paper is multi-fold. We provide a model and a solution algorithm for finding an optimal solution for the static-dynamic uncertainty strategy with penalty cost. In contrast to the related literature presented above, in which certainty equivalent mathematical programming formulations dominate, a dynamic programming

(DP) based approach is adopted. The optimal parameters of the replenishment policy is determined via exhaustive search, which is impractical for large instances. Therefore, we develop two heuristics: Approximation Heuristic (AH) and Relaxation Heuristic (RH). Our numerical experiments show that both heuristics perform well in terms of solution quality and computation time, and find the optimal policy over a wide range of model parameters. Any practical size instance can be solved to near-optimality using these heuristics. Finally, we discuss how our results and heuristics can be extended to handle capacity limitations and minimum order quantity considerations. The details of this study are reported in Özen et al. [7].

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Effective Network Models for the Two-Level Serial Lot Sizing Problems

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We consider the two-level lot sizing problems in serial supply chains where production, inventory, and shipment decisions are made centrally over a T-period planning horizon. A typical objective of these problems is to determine when and in what quantities to replenish (through either production or procurement), the inventories at each level, and when a downstream shipment should be made so that the external known demand at the final level is satisfied at a minimal total cost including fixed-charge replenishment/shipment costs and inventory holding costs at all levels. Several applications are encountered in practice especially for the two-level serial supply chains including, for instance, a warehouse and distribution center [5] or a supplier and manufacturer [9]. We study the uncapacitated problem where demand can be backlogged at the final level and the problem with capacitated shipments to the final level. In the following, we provide a brief overview of the literature that closely relate to the problems we address in this work.

For the uncapacitated L-level serial lot sizing problem without backlogging, Zangwill [13] presents a network representation and proposes a polynomial time DP algorithm based on the arborescent flows on the network. In an arborescent flow no node can have more than one positive input. This corresponds to the W-W property of an optimal solution to the single-level lot sizing problem with concave costs [12]. In most of what follows the W-W or ZIO (Zero-Inventory Ordering) property constitutes the foundation of the developed algorithms. Love [6] addresses the problem in [13] to show that if inventory holding costs are nondecreasing with respect to the production levels and production costs are nonincreasing in time, a nested schedule having the property that in a given period, production at level l implies production at level $l+1$, will be optimal. This nested structure leads to an algorithm that runs in $O(LT^3)$ time. Later, van Hoesel et al. [11] show that Zangwill's algorithm runs in $O(LT^4)$ time when $L > 2$ and in $O(T^3)$ time when $L = 2$. Recently, Melo and Wolsey [7] propose a new DP algorithm running in $O(T^2 logT)$ time for the two-level problem based on the observation that production batches at the second level are a refinement of those at the first.

van Hoesel et al. [11] also extend Zangwill's work by proposing polynomial DP algorithms for the more general problem that considers a stationary capacity on production at the initial level of the supply chain. For the general concave production, transportation, and inventory costs, the running time is $O(LT^{2L+3})$ which, for $L = 2$, reduces to $O(T^6)$ when inventory costs are linear and transportations costs are fixed-charge with no speculative motives. The two-level problem with fixed-charge and general concave transportation but linear production and inventory costs is studied in Kaminsky and Simchi-Levi [4]. Production is capacitated at the first level. They present a DP-based polynomial algorithm with $O(T^4)$ time complexity for the fixed-charge, nonstationary capacity problem. The complexity increases to $O(T^8)$ when transportation costs are general concave with stationary capacity. This is later improved to $O(T^7)$ in [11] as mentioned before. Considering nonconcave, stepwise fixed-charges for shipment costs, Lee et al. [5] present a shortest path network and propose an algorithm with a time complexity of $O(T^6)$ when backorders are allowed to better utilize the transportation scale economies. The case without backorders runs in $O(T^4)$ time. The shipment cost they consider includes a fixed cost and a freight cost that is a function of the used cargo capacity defined in terms of discrete increments. By identifying the regeneration periods (a period with zero ending inventory) for both levels of the problem, they define the structural properties of an optimal solution and develop DP algorithms. Sargut and Romeijn [9] study the two-level problem with backlogging at the second level and both capacitated and uncapacitated upstream sources also considering an outsourcing option. They provide a network representation and develop a solution methodology by generalizing the results from [4] and [11]. They provide solution procedures that run in $O(T^6)$ time for the capacitated and $O(T^7)$ time for the uncapacitated cases when there is no outsourcing. Assuming stationary fixed-charge transportation costs, linear inventory holding costs and no backlogging of demand, Jin and Muriel [3] develop an $O(T^3)$ algorithm for the single-retailer case and a nested shortest-path algorithm for the multi-retailer problem that runs in polynomial time for a given number of retailers. Variants of the two-level problem such as the problem that considers multiple modes of transportation [2] and the problem where the W-W property does not apply at the second level [10] have also been studied in the literature. Jaruphongsa et al. [2] suggest $O(T^4)$ and $O(T^5)$ algorithms considering alternative fixed cost delivery charges. Solyalı and Süral [10] consider an exogenous base-stock inventory at the second level such that when a shipment is made inventory has to be brought up to a specific order-up-to level. They propose an algorithm that runs in $O(T^3)$ time.

In this study, we consider two different two-level serial lot sizing problems and propose effective network models for them. The first problem we consider is the uncapacitated two-level serial lot sizing problem with backlogging (2L-U-B), where the backlogging of demand at the final level (retailer) is allowed and there are no capacities over the replenishments/shipments to the levels. The aim is to determine when and in what quantities to replenish at each level over the planning horizon, so that the sum of fixed-charge replenishment/shipment costs and inventory holding/backlogging costs at both levels is minimized. For 2L-U-B, making use of the regeneration intervals at the retailer [8] and the W-W property at the first level (warehouse), we propose an effective shortest path network representation, which can be solved in $O(T^3)$ time using a standard DP algorithm. Moreover, we present a tight formulation for 2L-U-B. The second problem we consider is the two-level serial lot sizing problem with cargo capacities and backlogging (2L-C-B). Similar to 2L-U-B, the backlogging of demand at the final level (retailer) is allowed and there is no capacity over the replenishments to the first level (warehouse). However, in 2L-C-B, we consider cargo capacities for shipments to the final level and the related stepwise fixed shipment costs as in [5]. The aim is to determine when and in what quantities to replenish at each level over the planning horizon, so that the sum of fixed-charge replenishment costs at the warehouse, stepwise fixed shipment costs at the retailer, and inventory holding/backlogging costs at both levels is minimized. By making use of the regeneration intervals at the retailer [1] and the W-W property at the warehouse, we propose a novel shortest path network representation for 2L-C-B. We show that this network can be solved in $O(T^5)$ time, which is an $O(T)$ improvement over the algorithm of Lee et al. [5].

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A Single Phase Dynamic Program with Independent Production Decision for Production-Capacitated Two- and Multi-Stage Lot-Sizing Problems

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1 Introduction

We consider a multi-stage capacitated lot-sizing problem (MLSP-PC), where the goal is to generate a centralized production and distribution plan for a supply chain which consists of a manufacturer with finite production capacity, intermediate agents and a retailer facing deterministic demand at the last stage. Clearly, centralized planning of an entire supply chain will lead to a lower-cost production and distribution plan than decentralized planning by independent agents. However, one of the obstacles to the centralized planning is the time to optimality due to the increased problem complexity from the simultaneous consideration of all problem parameters. Thus, a key to reducing this complexity is to allow decentralized or independent decisions of each agent whenever doing so will not negatively impact the overall solution. One of the purposes of this paper is to show that production decisions at the first stage of the supply chain can be made independently from transportation decisions at other stages in certain settings without negatively impacting the solution. This independence of the first-stage decision from other stages leads us to first focus on the two-stage problem (2LSP-PC) in which only two agents exist, the manufacturer and the retailer. The multi-stage problem with certain cost structures can be addressed in a similar fashion.

The single-stage uncapacitated lot-sizing problem for a manufacturer was introduced by Wagner and Whitin (1958), and efficient solution algorithms were designed by Federgruen and Tzur (1991), Wagelmans et al. (1992) and Aggarwal and Park (1993). The multi-stage version of the uncapacitated problem was solved by Zangwill (1969). To deal with the manufacturer's production capacity, Florian and Klein (1971) solved the capacitated single-stage lot-sizing problem (See also van Hoesel and Wagelmans 1996). Optimal algorithms for the multi-stage problem accounting for production capacity are provided by van Hoesel et al. (2005) and Hwang et al. (2011).

The MLSP-PC in general assumes concave production, transportation and inventory carrying costs through the planning horizon and the supply chain. As a special case of concave cost structure, the so-called *non-speculative* (transportation) cost structure assumes that inventory holding cost functions are linear and each of transportation cost functions consists of a fixed setup cost and per-unit transportation cost in which no speculative motive is allowed for keeping inventory. If each transportation does not incur setup cost then the supply chain is said to be have *linear* transportation costs. For the 2LSP-PC with the length of planning horizon T, Kaminsky and Simchi-Levi (2003) developed an $O(T^8)$ algorithm for their class of concave transportation costs and van Hoesel et al. (2005) presented three algorithms with complexities $O(T^7)$, $O(T^6)$ and $O(T^5)$ for concave, non-speculative transportation and linear transportation cost structures, respectively. Van Hoesel et al. (2005) also put the complexity reduction of their algorithms as an open question. In this paper, we address this question by deriving $O(T^6)$, $O(T^5)$ and $O(T^4)$ algorithms for concave, non-speculative transportation and linear transportation cost structures in the 2LSP-PC, respectively. For the multi-stage production capacitated problem with non-speculative costs, we present an efficient $O(T^6)$ algorithm as compared to the $O(T^7)$ algorithm of van Hoesel et al. (2005).

We note that all the algorithms in this paper are efficient by a factor of $O(T)$ over the best known ones until now. Most of these improvements are made by making possible separating the production decision from transportation decisions. To support such independence, it is crucial to have an appropriate dynamic programming algorithm. We adapt the single-phase dynamic programming approach first developed in Hwang et al. (2011) for multi-stage problems.

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The Single-Item Green Lot-Sizing Problem with Fixed Carbon Emissions

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1 Introduction

In this work, we investigate the multi-sourcing single-item lot-sizing problems with fixed and variable carbon emissions. This problem is an extension of the lot-sizing problem with periodic carbon emission constraints introduced in [1] where only variable carbon emissions were considered.

The multi-sourcing single-item lot-sizing problem can be defined as the problem faced by a company that has to determine, over a planning horizon of T periods, when, where and how much to produce an item to satisfy a deterministic time-dependent demand. Different production locations and transportation modes are available to satisfy a given demand. We consider M different supplying modes, where a supplying mode is defined as the combination of a production location and a transportation mode (combining one or more types of vehicles). There are no capacity constraints, but we consider periodic carbon emission constraints with fixed and variable carbon emissions. The carbon emission of each supplying mode is modeled using a linear function of the supplied quantity plus a fixed carbon emission. The fixed carbon emission is incurred is incurred at each period the mode is selected. The objective function consists in minimizing the total supplying costs (fixed and variable) as well as the inventory costs.

In this work, we propose some structural properties of dominant solutions and show that our problem can be solved using a dynamic programming algorithm when carbon emission parameters are stationary. In contrast, we prove that the problem with time varying carbon emission parameters is $\mathcal{NP}$-hard.

2 Mathematical model

To model the multi-sourcing lot-sizing problem with fixed and variable carbon emissions, the following parameters and variables are required.

Parameters:
d_t: Demand at period t,
$h_t(s)$: Cost of holding s units at the end of period t,
p_t^m: Unitary supplying cost of mode m at period t,
f_t^m: Supplying setup cost of mode m in period t,
ev_t^m: Environmental impact (carbon emission) related to supplying one unit using mode m at period t,
ef_t^m: Fixed environmental impact (carbon emission) related to using mode m at period t,
$E_t^{\max}$: Maximum unitary environmental impact allowed at period t.

Variables:
x_t^m: Quantity supplied at period t using mode m,
y_t^m: Binary variable which is equal to 1 if mode m is used at period t, and 0 otherwise,
s_t: Inventory carried from period t to period $t+1$.

The formulation for the multi-sourcing single-item lot-sizing problem with periodic carbon emission constraints is given below:

$$\min \quad \sum_{m=1}^{M} \sum_{t=1}^{T} (p_t^m x_t^m + f_t^m y_t^m) + \sum_{t=1}^{T} h_t(s_t) \tag{1}$$

$$s.t. \quad \textstyle\sum_{m=1}^{M} x_t^m - s_t + s_{t-1} = d_t \qquad t = 1, \ldots, T \tag{2}$$

$$\textstyle\sum_{m=1}^{M} (ev_t^m - E_t^{\max}) x_t^m + ef_t^m y_t^m \leq 0 \quad t = 1, \ldots, T \tag{3}$$

$$x_t^m \leq \left(\textstyle\sum_{t'=t}^{T} d_{t'} \right) y_t^m \qquad t = 1, \ldots, T, m = 1, \ldots, M \tag{4}$$

$$x_t^m \in \mathbb{R}^+, y_t^m \in \{0, 1\} \qquad t = 1, \ldots, T, m = 1, \ldots, M \tag{5}$$

$$s_t \in \mathbb{R}^+, \qquad t = 1, \ldots, T \tag{6}$$

The objective function (1) minimizes the fixed and variable production and transportation costs and holding costs. Constraint (2) models flow conservation, and Constraint (3) forces the average amount of carbon emission at any period t to be lower or equal than the maximum unitary environmental impact allowed. Constraint (4) ensures that no item can be supplied with mode m at period t if this mode is not selected.

3 Complexity results

In what follows, we establish some dominance properties for the Uncapacitated Lot-Sizing problem with the Periodic Carbon emission constraint, and fixed and variable carbon emissions (ULS-PC-F). We then show that ULS-PC-F can be solved using a dynamic program if carbon emissions are stationary and is $\mathcal{NP}$-Hard if these parameters are non stationary. For sake of conciseness, we denote by $\bar{e}_t^m$ the expression $(ev_t^m - E_t^{\max})$. A mode m is called *ecological* or *environmental friendly* in period t if $\bar{e}_t^m \leq 0$. Note that a non-ecological mode cannot be used before fixed carbon emissions are compensated. If a mode m_1 (or a combination of modes m_1 and m_2) is used to produce a given demand, a minimal production threshold $Q_t^{m_1}$ (or $Q_t^{m_1 m_2}$) is required to compensate the associated fixed carbon emissions.

Property 2 *Consider a solution of the ULS-PC-F problem using two modes m_1 and m_2 in a given period t. If $p_t^{m_1} \leq p_t^{m_2}$ and $ev_t^{m_1} \leq ev_t^{m_2}$, then mode m_1 dominates mode m_2 (i.e. a solution using modes m_1 and m_2 can be improved by removing the quantity $x_t^{m_2}$ produced using mode m_2 and by producing $x_t^{m_2}$ using mode m_1).*

Property 3 *Any solution of the ULS-PC-F problem uses at least one ecological mode in each period with an order, i.e a strictly positive supplied quantity.*

Theorem 1 *There exists an optimal solution for the ULS-PC-F problem that uses at most two modes in each ordering period: One ecological mode and possibly one non-ecological mode.*

Lemma 1 *For the ULS-PC-F problem, the cost of the best ZIO (Zero-Inventory-Ordering) policy may be arbitrary large compared to the cost of an optimal policy.*

Definition 1 *(ordering period)*

- *A period t is a* ***threshold ordering period*** *if there is a mode m such that $x_t^m = Q_t^m$.*
- *A period t is an* ***ordering period*** *if there is a mode m such that $x_t^m > Q_t^m$.*

Definition 2 *(Regeneration point) A period t is an* ***regeneration point*** *if ($s_t = 0$).*

A dominant solution has the following properties.

Property 4 *Between two consecutive regeneration points, there exists at most one ordering period.*

Property 5 *All threshold ordering periods occur before the production ordering period.*

From these properties, we can show the following results:

Theorem 2 *The ULS-PC-F with a fixed number of modes M and stationary carbon emissions can be solved using a dynamic programming algorithm.*

Theorem 3 *ULS-PC-F is $\mathcal{NP}$-Hard if carbon emission parameters are not stationary.*

The theorem above can be proved by performing a reduction from the PARTITION problem.

4 Perspectives

In this work, carbon emissions are aggregated in each supplying mode (a combination of a production location and a transportation mode, requiring one or more types of vehicles). The carbon emission of each supplying mode is modeled using a linear function of the delivered quantity plus a fixed carbon emission. This model could be detailed by defining a fixed carbon emission that depends on the number of required "vehicles" (e.g. containers). It would be interesting to study the added value of a more detailed carbon emission constraint and the complexity of the resulting problems.

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Bi-Objective Economic Lot-Sizing problems

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Abstract
Nowadays companies try to reduce their carbon footprint. Bearing in mind this environmental awareness, the choice of a production plan can be modeled as a Bi-Objective Economic Lot-Sizing problem, in which we aim to minimize the total lot-sizing costs, as well as minimizing the maximum emission across blocks of given length. In this talk, we first show that finding a single Pareto efficient outcome is, in general, an $\mathcal{NP}$-hard task. We then identify non-trivial classes of problem instances for which this problem is polynomially solvable. We end by showing how our results can be used to approximate the Pareto frontier.
Keywords: carbon emissions, lot-sizing, bi-objective, Pareto efficient outcomes

1 The outline of the talk

While over the last two decades many quantitative models have been developed for the design and management of *green* supply chains, [2, 11, 16], only very recent papers have addressed the carbon footprint of the supply chain, see [3, 6, 7, 8, 9, 10, 13, 14, 17, 18]. Benjaafar et al. [4] are one of the first to include carbon emissions in the classical *Economic Lot-Sizing (ELS) Problem*, by means of emission caps, taxes on emissions, cap-and-trade emission mechanisms, or carbon offsets. See also [12] for a lot-sizing model with an ecoterm in the objective function. More recently, [15] and [1] have provided theoretical and algorithmic results on lot-sizing models with emission constraints.

Bearing in mind the current environmental awareness, we model the choice of a production plan as a Bi-Objective Economic Lot-Sizing Problem. Consider a planning horizon of length T. For period t ($t = 1, \ldots, T$), let f_t be the setup lot-sizing cost, c_t the unit production lot-sizing cost and h_t the unit inventory holding lot-sizing cost. Similarly, for period t, let $\hat{f}_t$ be the setup emission, $\hat{c}_t$ the unit production emission and $\hat{h}_t$ the unit inventory emission. Let d_t be the demand in period t. Let us partition the time horizon into consecutive blocks of ℓ periods. The Bi-Objective Economic Lot-Sizing (BOLS$^{(\ell)}$) model with block size ℓ reads as follows:

$$\text{minimize} \left(\sum_{t=1}^{T} [f_t y_t + c_t x_t + h_t I_t], \max_{i=1,\ldots,T/\ell} \sum_{t=(i-1)\ell+1}^{i\ell} [\hat{f}_t y_t + \hat{c}_t x_t + \hat{h}_t I_t] \right)$$

subject to (BOLS$^{(\ell)}$)

$$\begin{array}{rcll} x_t + I_{t-1} & = & d_t + I_t & t = 1, \ldots, T \\ x_t & \leq & My_t & t = 1, \ldots, T \\ I_0 & = & 0 & \\ y_t & \in & \{0, 1\} & t = 1, \ldots, T \\ x_t & \geq & 0 & t = 1, \ldots, T \\ I_t & \geq & 0 & t = 1, \ldots, T. \end{array}$$

If more than one objective function is optimized, Pareto efficient outcomes (in the value space) are sought. These can be found by minimizing one objective function while constraining the others. Given $\hat{b} \in \mathbb{R}_+$, the following problem defines a Pareto efficient outcome for (BOLS$^{(\ell)}$):

$$\text{minimize} \sum_{t=1}^{T} [f_t y_t + c_t x_t + h_t I_t]$$

subject to ($\mathcal{P}^{(\ell)}(\hat{b})$)

$$\begin{array}{rcll} x_t + I_{t-1} & = & d_t + I_t & t = 1, \ldots, T \\ x_t & \leq & My_t & t = 1, \ldots, T \\ I_0 & = & 0 & \\ y_t & \in & \{0, 1\} & t = 1, \ldots, T \\ x_t & \geq & 0 & t = 1, \ldots, T \\ I_t & \geq & 0 & t = 1, \ldots, T \\ \sum_{t=(i-1)\ell+1}^{i\ell} [\hat{f}_t y_t + \hat{c}_t x_t + \hat{h}_t I_t] & \leq & \hat{b} & i = 1, \ldots, T/\ell. \end{array}$$

In this talk, we first show that finding a single Pareto efficient outcome is, in general, an $\mathcal{NP}$-hard task. We then identify classes of instances for which ($\mathcal{P}^{(\ell)}(\hat{b})$) is polynomially solvable. In particular, we analyze classes for which the costs are non-speculative, and the setup and production emissions are time-invariant. A summary of our results is given below. We end by showing how our results can be used to find an ε-dominating set in the outcome space, see [5] and references therein.

ℓ	costs	emissions	running time
T	$f_t = f$, $c_t = c$	$\hat{f}_t = \hat{f}$, $\hat{c}_t = \hat{c}$, $\hat{h}_t = \alpha h_t$	$\mathcal{O}(T^2)$
1	$f_t \geq f_{t+1}$, non-speculative	$\hat{f}_t = \hat{f}$, $\hat{c}_t = \hat{c}$, $\hat{h}_t = \hat{h}$	$\mathcal{O}(T^2)$
fixed ℓ	non-speculative	$\hat{c}_t = 0$	$\mathcal{O}(T^2)$
general ℓ	non-speculative	$\hat{f}_t = \hat{f}$, $\hat{c}_t = 0$, $\hat{h}_t = 0$	$\mathcal{O}(\ell T^2)$
general ℓ	non-speculative	$\hat{f}_t = \hat{f}$, $\hat{c}_t = \hat{c}$, $\hat{h}_t = 0$	$\mathcal{O}(T^7/\ell)$
general ℓ	non-speculative	$\hat{f}_t = 0$, $\hat{c}_t = \hat{c}$, $\hat{h}_t = 0$	$\mathcal{O}(T^5)$

Table 1: Polynomially solvable cases of $\mathcal{P}^{(\ell)}(\hat{b})$

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